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MATHEMATICS
OF
Air and Marine
NAVIGATION

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PREFACE

The purpose of this text is twofold — to provide the future navigator with a substantial foundation of mathematical theory, and to acquaint the interested layman with some of the problems encountered in navigating aircraft and surface vessels. It may be successfully studied by anyone who has completed the usual high school course in plane trigonometry.

The treatment is unusually concrete and direct. Numerous diagrams make the explanations easy to follow. Although the essential parts of spherical trigonometry have been carefully developed, the student has not been burdened with unnecessary formulas nor with all the cases of triangle solution. Attention is called to the use of haversines, which simplify both formulas and computations.

Five-place tables have been provided for the solution of numerical problems. Since the data are generally given to four significant figures and the nearest minute of angle, no interpolation is required except with the haversine table. Many of the problems are solved by graphical construction; with these not more than three-place accuracy can be expected. A few problems require special charts or reference to the *Nautical Almanac* for their solution.

References to other books and to maps and charts are given at the end of each chapter. The following books are mentioned in the bibliographies by author's name only:

Dutton, Benjamin, *Navigation and Nautical Astronomy*, U. S. Naval Institute, Annapolis, Md., 1942.

Bowditch, Nathaniel, *American Practical Navigator*, U. S. Hydrographic Office, 1938.

Aircraft Navigation Manual, U. S. Hydrographic Office Publication, No. 216. Referred to as H. O. 216.

Deets and Adams, *Elements of Map Projection*, U. S. Coast and Geodetic Survey, 1928.

Lyon, T. C., *Practical Air Navigation*, Civil Aeronautics Bulletin, 1940.

The author wishes to thank Professor Clifford B. Upton of Teachers College, Columbia University, for valuable advice and criticism. He has also received numerous helpful suggestions from members of his classes. And finally he wishes to acknowledge the assistance of Gladys M. Bradley in the preparation of the manuscript.

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CHAPTER I

GEOMETRY OF THE EARTH

The navigator of an airplane or a ship must solve two problems: One is to determine the direction and distance of one point from another so that he may "chart his course"; the second is to determine the position of his plane or ship at frequent intervals so that he may know whether he is actually following the desired route. Chapter I will explain some of the principles of geometry which enter into these two problems.

Shape and Size of the Earth. It has long been known that the earth is nearly spherical in form. This conclusion was reached by different lines of reasoning. As a ship at sea approaches land, an observer on the ship sees the highest points on the land first. Similarly, when a ship sails away, the lower part is seen to sink from sight first and then finally the top part of the ship disappears. Ships and planes have traveled around the earth and returned to their starting point. When an observer looks across the earth's surface, the earth and sky appear to meet in a circle called the horizon. A perfect horizon may be seen from a ship or plane or possibly from a tall building. The first photograph showing the curve of the horizon was made in 1935 by Stevens and Anderson from a record height of 13.71 miles.

Reasons like the ones given above do not prove that the earth is spherical but show merely that it is a curved surface. Experiments and measurements have shown that the earth curves nearly the same in all directions. Scientists have devoted much time and effort to finding the exact size and shape of the earth. Careful measurements have shown that the distance from the earth's center to the poles is approximately 3950 miles and that the radius at the equator is 3963 miles. This difference of 13 miles is relatively very small in comparison with the distances of nearly four thousand miles from the center; therefore for all purposes of navigation the earth is considered to be a sphere.

For ordinary purposes, distances are measured in the usual mile of 5280 feet. The mile of 5280 feet is really a *land mile* or *statute*

mile, and must be distinguished from a different unit called the *nautical mile* which is used by the navigator. The length of our common mile was not derived from measuring the earth but probably has come down to us from the thousand paces (*millia passuum*) of the Roman soldier. The nautical mile used by navigators is the length of one minute of the earth's circumference. Since the earth's circumference contains 360° or $21,600'$, it also contains 21,600 nautical miles. The length of one minute of the earth's circumference depends on whether it is measured along the equator or on a circumference passing through the poles. This variation is quite small and the standard nautical mile adopted by the United States Government is 6080.27 feet.

Problems

1. Show that 1 nautical mile = 1.152 statute miles, and that 1 statute mile = .8684 nautical mile.
2. Using the formula that the circumference of a circle equals 2π times the radius ($C = 2\pi r$), find the radius of the earth in nautical and statute miles.
3. Find the earth's circumference in statute miles.
4. The area of a sphere is $4\pi r^2$. Find the area of the earth in square statute miles and square nautical miles.
5. The volume of a sphere is $\frac{4}{3}\pi r^3$. Find the volume of the earth in cubic statute miles and cubic nautical miles.

Earth Co-ordinates. There are many ways of describing the location of an aircraft or a ship or any point on the earth. The position of a plane may be given as ten miles northeast of LaGuardia Field, or a ship may be located by saying it is two miles southeast of the Ambrose Channel Lightship. Statements such as these cannot be used to locate a plane flying over the ocean or to give the position of a ship which is beyond the sight of land. In many situations, therefore, the navigator must state his position in terms of earth co-ordinates which are called latitude and longitude.

Latitude and longitude correspond rather closely to the x, y co-ordinates which are used in algebra. The earth co-ordinates, however, are measured in degrees and minutes of angle instead of some measure of length such as miles or feet.

It is known from geography that the earth rotates or spins once each day about a diameter which joins the north and south poles. The circle around the earth midway between the poles is called the equator. In other words, the equator is a circle which lies in a plane

perpendicular to the polar diameter. Figure 1 is a cross section of the earth through the north and south poles P and P' ; EE' is a diameter of the equator; and AA' is a line parallel to the equator with the arc EA equal to 40° . If Figure 1 is rotated about the line PP' , then the line EE' will trace out the equator and AA' a smaller circle parallel to the equator as shown in Figure 2.

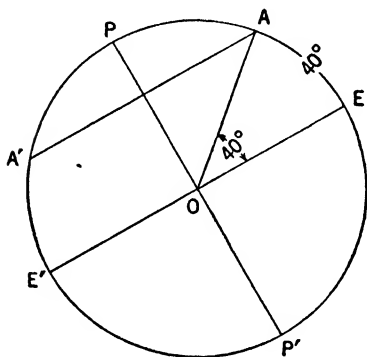


FIGURE 1.

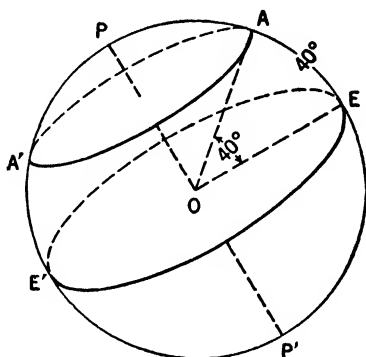


FIGURE 2.

The circle AA' of Figure 2 is called a parallel of latitude and all the points on it are said to have a latitude of 40° north (40° N.). The latitude of 40° N. is measured either by the central angle AOE or by the corresponding arc on the circumference of the earth, AE .

All points on the equator have a latitude of 0° . Latitude varies from 90° S. at the south pole to 90° N. at the north pole. South latitudes are sometimes called negative. The equator corresponds to the X axis or $y = 0$, and the parallels of latitude to the lines parallel to the X axis or $y = 1, y = 2$, etc. Latitude alone does not fix the position of a point, except in the case of the north and south poles, any more than $y = 2$ fixes a point.

In Figure 3, $PEP'E'$ and $PBP'B'$ are two circles formed by taking cross sections of the earth through the polar diameter. The semicircles like PBP' and PEP' are called *meridians*. Let PEP' be the meridian of Greenwich, England (latitude $51^\circ 29'$ N.), and let EB be an arc of the equator

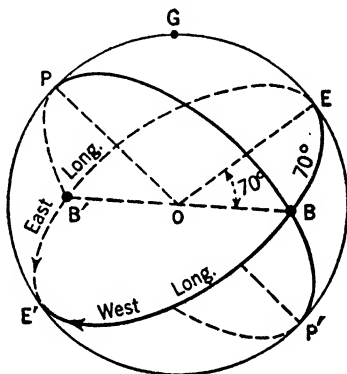


FIGURE 3.

equal to 70° . All points on PEP' , the meridian of Greenwich, have a longitude equal to 0° , and all points on the meridian PBP' have a longitude of 70° west (70° W.). Longitude is measured east or west to 180° from the meridian of Greenwich. Since the arc EB and the angle EOB equal 70° , the arc EB' and the angle EOB' equal 110° ; hence all points on $PB'P'$ are in longitude 110° E. The meridian $PE'P'$ contains all points of longitude 180° .

The meridian of 0° corresponds to the Y axis and the other meridian to the lines $x = 1$, $x = 2$, etc., although the meridians meet at the poles. In Figure 3, longitude was seen to be measured either by an arc of the equator or an angle at the earth's center. Longitude may also be measured along any parallel.

Latitude is the angular distance north or south of the equator measured along the arc of a meridian.

Longitude is the angular distance east or west of the meridian of Greenwich measured along the equator or along a parallel.

It is quite natural to measure latitude north or south from the equator, since the equator is the largest parallel of latitude. The meridians are all semicircles of the same size, and any meridian might have been chosen as the prime meridian. The meridian of Greenwich was adopted by the English after the establishment of the Observatory there and has since been adopted by most countries. It may be interesting to note that the meridian of Washington has sometimes been used as a reference meridian by our Government. In 1864 Congress passed an act stating that the boundaries of Colorado should be the parallels of latitude of 37° N. and 41° N. and the meridians of 25° and 32° west from Washington.

Problems

1. Find the latitude and longitude of your school by reference to the best map available.

2. Take as many maps of the world as are available and find the interval in degrees between successive meridians and parallels.

3. The latitudes and longitudes given below locate six important cities. Identify the cities from a map or globe.

Latitude:

$38^\circ 55'$ N. $33^\circ 52'$ S. $51^\circ 29'$ N. $34^\circ 40'$ N. $55^\circ 45'$ N. $22^\circ 54'$ S.

Longitude:

$77^\circ 04'$ W. $151^\circ 12'$ E. $0^\circ 0'$ $135^\circ 11'$ E. $37^\circ 34'$ E. $43^\circ 10'$ W.

4. Find from a globe or map the approximate latitude and longitude of the following places: Cairo, Valparaiso, Singapore, Melbourne, Cologne.

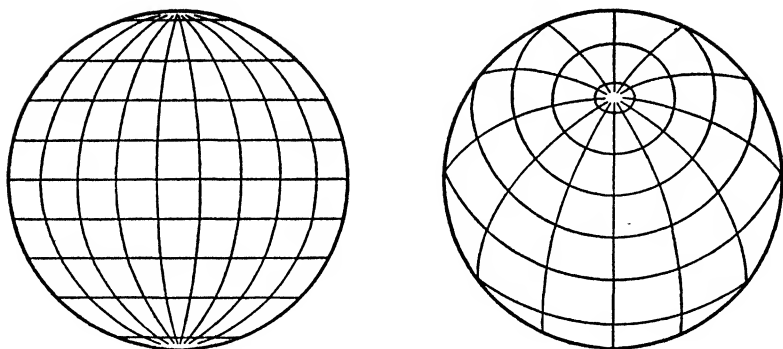


FIGURE 4.

Great Circles on the Earth. The parallels of latitude and the meridians form a network of intersecting circles on the surface of the earth. The parallels decrease in size toward the poles, while the meridians are all the same size. The equator and the meridians are examples of *great circles*.

A great circle on the earth or any sphere is formed by the intersection of the sphere and a plane which passes through its center.

There is one great circle through any two points on a sphere, since three points determine a plane. Great circles are important in navigation because of the following theorem:

The shortest distance between two points on the surface of a sphere is the shorter arc of the great circle which joins them.

The great circle between two points on the earth may be easily indicated on a terrestrial globe. Take a square of stiff cardboard and at its center construct a circle equal to the equator of the globe, cut out the circle and mark off degrees around its circumference as in Figure 5. This cardboard may be used as a "great circle protractor" to find the approximate number of degrees in the great circle arc between two points. It will also indicate the

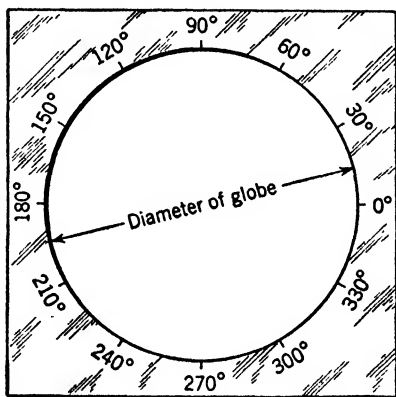


FIGURE 5.

two points where the great circle intersects the equator and the two points nearest the poles. For example, the great circle arc between San Francisco and Tokyo has a length of approximately 74° or 4440 nautical miles and the point nearest the north pole is about 49° N., 170° W. If this great circle arc is extended, it will cross the equator at longitude 80° W. and 100° E.

The accuracy of the answers obtained by this method will depend upon the size of the globe which is available and the accuracy of the cardboard protractor. In a later chapter a method of finding great circle distances by trigonometry will be developed.

The cardboard protractor and the equator of the globe illustrate another theorem.

Two great circles intersect in two points which are the ends of a diameter of the sphere.

The meridians also illustrate this theorem.

Problems

Find the approximate great circle distances in degrees and nautical miles between the following points.

1. Natal ($5^\circ 47'$ S., $35^\circ 13'$ W.) to Dakar ($14^\circ 40'$ N., $17^\circ 25'$ W.)
2. New York ($40^\circ 43'$ N., $74^\circ 00'$ W.) to Cape of Good Hope ($34^\circ 21'$ S., $18^\circ 29'$ E.)
3. Sydney ($33^\circ 52'$ S., $151^\circ 12'$ E.) to Valparaiso ($33^\circ 01'$ S., $71^\circ 39'$ W.)
4. Auckland ($36^\circ 50'$ S., $174^\circ 46'$ E.) to Sitka ($57^\circ 03'$ N., $135^\circ 20'$ W.)
5. Harbour Grace ($47^\circ 43'$ N., $53^\circ 08'$ W.) to Moscow ($55^\circ 45'$ N., $37^\circ 34'$ E.)
6. Curtiss Field ($40^\circ 45'$ N., $73^\circ 37'$ W.) to Paris ($48^\circ 50'$ N., $2^\circ 20'$ E.)

Small Circles on the Earth. In later problems it will be necessary to consider *small circles* on the earth. The parallels of latitude are examples of small circles. In general every plane will intersect a sphere in a circle. If the plane passes through the center of the sphere, the intersection of the plane and the sphere is a great circle. If the plane does not pass through the center of the sphere, the intersection is called a small circle. Figure 6 shows a plane intersecting a sphere in a small circle.

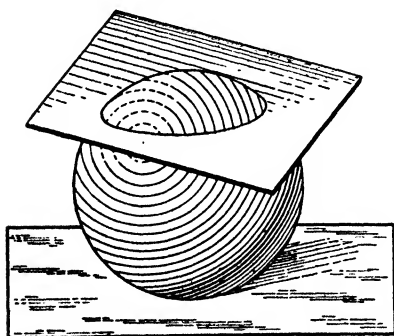


FIGURE 6.

All the points on the parallel of 80° N. are 10° or 600 nautical miles from the north pole. If the parallels were not marked on a globe, they could easily be constructed in much the same way that circles are drawn with the compasses in a plane. To construct the parallel of 80° N. one would place one point of the compasses at 0° longitude on the equator, and the other point at 10° W. on the equator. Then using this "radius" the parallel could be drawn on the globe with the north pole as "center." In a similar way any parallel might be drawn.

Suppose that a small circle equal to the parallel of 80° N. is drawn with Washington, D. C., as "center"; this circle will contain all the points on the earth which are 600 nautical miles or 10° distant from Washington. Washington is said to be one of the *poles* of this small circle and the 10° distance is called the polar distance

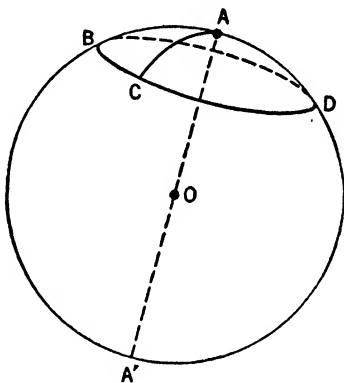


FIGURE 7.

of the circle. The definitions of poles and polar distances of a small circle are:

The poles of a small circle are the ends of the diameter of the sphere which is perpendicular to the plane of the small circle.

The polar distance of a circle of a sphere is the arc of a great circle from the nearer pole to any point on the small circle.

In Figure 7, A and A' are the poles of the small circle BCD; and the polar distance of the small circle is measured by any of the great circle arcs AB, AC, etc. The polar distance of any parallel of latitude is the complement of the latitude.

Problems

1. Construct on a globe small circles of polar distances of 10° , 20° , and 30° which have Chicago ($41^{\circ} 50' \text{ N.}$, $87^{\circ} 37' \text{ W.}$) as a pole.
2. Construct the small circles which have Halifax ($44^{\circ} 40' \text{ N.}$, $63^{\circ} 35' \text{ W.}$) as a pole and which have polar distances of 315, 430, 635 nautical miles.
3. Construct a small circle with Lima ($12^{\circ} 03' \text{ S.}$, $77^{\circ} 00' \text{ W.}$) for a pole and a polar distance of 30° . Find the intersections of this circle with a small

circle which has New York ($40^{\circ} 43' \text{ N.}$, $74^{\circ} 00' \text{ W.}$) for a pole and a polar distance of 40° .

4. Find the longitudes at which the small circle with New York for pole and polar distance of 40° crosses the parallels of 10° N. , 20° N. , 30° N. , 40° N.

5. Find the latitudes at which the small circle with Lima for pole and polar distance of 30° crosses the meridians of 60° W. , 75° W. , 90° W. , and 105° W.

Triangles on the Earth. In plane trigonometry one studies various rules for solving plane triangles, that is, for finding certain sides and angles when certain other sides and angles are given. Navigation, like plane surveying, makes use of plane trigonometry. Plane trigonometry, however, does not make any allowance for the curvature of the earth and, when the distances involved are large, it is necessary to work with triangles on the surface of a sphere. A spherical triangle is a part of the surface of a sphere which is bounded by three great circles.

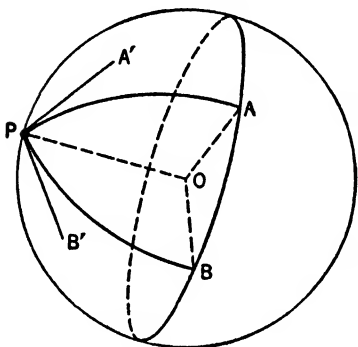


FIGURE 8.

Consider a spherical triangle PAB in Figure 8. It is formed by the arcs of three great circles, the half meridians PA and PB, and the arc of the equator AB. Since the sides of the triangle are arcs of great circles, they can be measured in degrees. PA and PB are each equal to 90° . AB can be found from the longitudes of points A and B. If PA is the meridian of 75° W. and PB the meridian of 120° W. , then the arc $AB = 45^{\circ}$. PAB is a special case of a spherical triangle since two of its sides are quadrants.

The angle between two great circles, commonly called a *spherical angle*, is defined as the angle between the tangents to the circles at their point of intersection. In Figure 8, angle P, or the angle between PA and PB, is, by definition, the angle between the tangents to PA and PB or angle $A'PB'$. Notice that the angles $A'PB'$ and AOB are equal because their sides are parallel; therefore angle P in the triangle is equal to the arc AB or 45° . The angles A and B in the triangle are each equal to 90° . Hence the sum of the angles of PAB is 225° .

Suppose that PB moves westward, then angle P increases and the sum of the three angles increases. If B moves eastward toward A,

the sum of the three angles decreases. This illustrates an important theorem.

The sum of the angles of a triangle formed by the arcs of three great circles is greater than 180° and less than 540° .

In the previous example two vertices of the spherical triangle were on the equator. Consider now a triangle formed on the earth by the meridian arcs from the north pole to San Francisco ($37^\circ 47' \text{ N.}$, $122^\circ 26' \text{ W.}$) and to Tahiti ($17^\circ 30' \text{ S.}$, $149^\circ 30' \text{ W.}$) and the great circle arc which joins these two places (see Figure 9).

The side $PS = 90^\circ - 37^\circ 47' = 52^\circ 13'$.

The side $PT = 90^\circ + 17^\circ 30' = 107^\circ 30'$.

Angle $P = 149^\circ 30' - 122^\circ 26' = 27^\circ 04'$.

Construct this triangle on a globe and determine approximately the length of the arc ST . Estimate the angles, and verify that their sum is greater than 180° .

In conclusion, the sides and angles of spherical triangles are both measured in degrees and minutes, and the sum of the angles is always greater than 180° . Spherical triangles like the triangle PST occur in many of the problems of navigation.

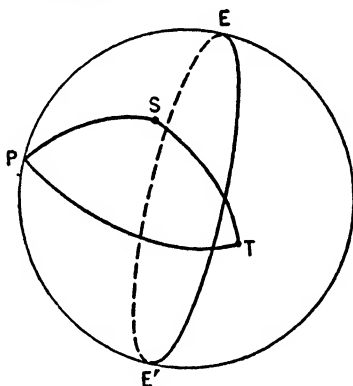


FIGURE 9.

Problems

For each of the problems on page 6, construct the triangle formed by the north pole and the two given points. Find the exact values of the angle at the pole and its two including sides. Estimate the other two angles and verify that the sum of the angles is greater than 180° .

Mapping the Earth. The problem of constructing accurate maps of part or all of the earth's surface has engaged man's attention from the earliest times. The Babylonians drew maps on clay tablets, and it is said that Rameses II had a map of the estates along the Nile in 1300 B.C. The early Polynesian navigators made maps of the Pacific to show the location of islands and the directions of winds and currents. Maps made during medieval times were naturally incomplete, and unexplored parts of the world were filled in with fantastic pictures of strange men and beasts.

There is some interest in comparing early maps of the New World

with our present-day maps. A "Map of New York I. with adjacent Rocks and other remarkable Parts of Hell Gate" drawn in 1778 by the British Hydrographer has an error of 1' in the latitude and 14' in the longitude.

Several departments of the United States Government are directly concerned with map making. The United States Geological Survey is engaged in preparing a large-scale topographic map of the entire country. The United States Hydrographic Office supplies the charts necessary in navigation and together with the British Admiralty has issued most of the navigation charts in general use. For 120 years the United States Coast and Geodetic Survey has been issuing charts of harbors and coasts, which show buoys, lights, shoals, channel depths, and prominent landmarks. The United States Department of Agriculture publishes large-scale maps which indicate the types of soil to be found in a given locality. No maps approach the detail of those of the British Ordnance Survey, which has prepared a map of the United Kingdom on a scale of twenty-five inches to the mile.

There are two problems in map making. One is to construct a network of lines to represent the meridians and parallels. The second is to locate coast lines, islands, rivers and all the other details at their proper positions on the network.

In a sense the problem of mapping the earth is the same as making a baseball cover. Examine a baseball closely and notice that at certain places the cover is slightly wrinkled and at others it is slightly stretched. The stretching and wrinkling occur because the surface of a sphere cannot be flattened out in a plane without distorting it in some way. This difficulty does not occur with the cone and cylinder. The surface of a right cone may be formed from the sector of a circle and the surface of a right cylinder from a rectangle (see Figure 10).

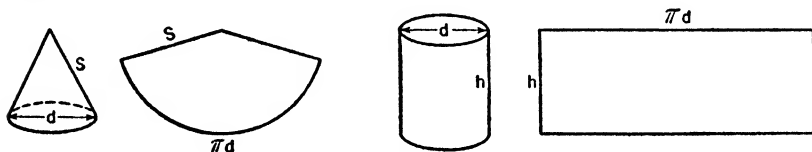


FIGURE 10.

Many different types of world maps have been developed for special purposes. In some maps the meridians are parallel lines and "the top of the map is north." Other maps are centered on the north pole and show the meridians as straight lines passing through this

point; in such maps north is toward the center of the map. On some maps both the meridians and parallels are straight lines, and on others either the meridians or parallels or both may be curves.

Problems

1. Take the world maps in an atlas or geography and compare the meridians and parallels. Notice whether the meridians pass through a common point and whether they are straight lines or curves. Notice whether the parallels have the same length or whether they become smaller toward the pole.

2. Make a similar comparison for maps of the Polar Regions.

Mercator's Map of the World. One of the most common world maps is called *Mercator's Projection*; in fact, most maps of the world



FIGURE 11.

are drawn on Mercator's plan. In the Mercator map the meridians are parallel straight lines, equal distances apart; the parallels of latitude are also parallel lines, but the distance between the parallels increases toward the poles.

It is worth while to compare the Mercator map with the globe. All the parallels on the map have the same length; on the globe of course the parallels become shorter toward the poles. At latitude 60° on the map, distances in both the east-west and north-south directions are doubled in length. Hence an island at latitude 60° is four times as large as it should be. Figure 11 shows how areas are enlarged by the Mercator map. Egypt, Anglo-Egyptian Sudan, and the Belgian Congo have been placed on the map of Greenland. Greenland has an area of only 827,300 square miles, while the combined area of the other three territories is 2,276,500 square miles. This shows that the Mercator map should not be used for comparison of areas.

Mercator's map has one important property which makes it important for navigation. Any two points on the earth may be connected by a curve which does not change direction; such a curve is called a *rhumb line*. A rhumb line is not to be confused with a great circle. The great circle between two points is the shortest distance between

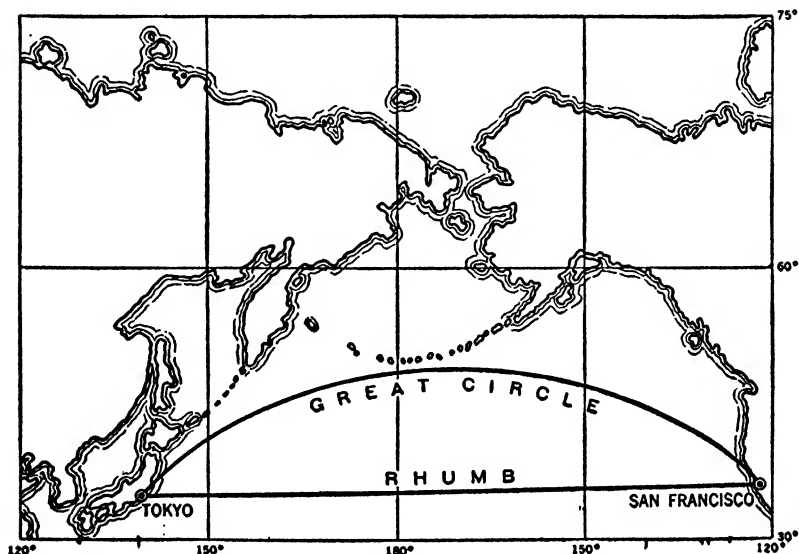


FIGURE 12. RHUMB LINE AND GREAT CIRCLE FROM SAN FRANCISCO TO TOKYO.

them, but as one follows a great circle, his direction will change from point to point as he moves along the arc. In place of the great circle one might follow the rhumb line between two points; in this case the direction would remain the same, but the total distance traveled would be longer. Only when two places are on the equator or on the same meridian will the rhumb line and the great circle between them coincide.

Figure 12 shows the rhumb line and the great circle between San Francisco and Tokyo plotted on the Mercator map. Notice that the rhumb line is a straight line on the map and the great circle is a curve, and hence *appears* to be longer than the rhumb line. Actually the rhumb line is 266 nautical miles longer than the great circle. Notice also that the great circle changes direction since it goes as far north as latitude 49° .

The importance of maps cannot be overemphasized. However, a true conception of areas and distances on the earth can be gained only from a globe.

Problems

In each of these problems a series of points on a great circle is given. Plot the great circles on a Mercator map and draw the corresponding rhumb lines.

1. Curtis Field ($40^{\circ} 45' \text{ N.}$, $73^{\circ} 37' \text{ W.}$) to Paris ($48^{\circ} 50' \text{ N.}$, $2^{\circ} 20' \text{ E.}$)

Latitude:	$46^{\circ} 50' \text{ N.}$	$51^{\circ} 30' \text{ N.}$	$52^{\circ} 10' \text{ N.}$	$49^{\circ} 30' \text{ N.}$
Longitude:	60° W.	40° W.	20° W.	0° W.

2. Cape Race ($46^{\circ} 39' \text{ N.}$, $53^{\circ} 04' \text{ W.}$) to Cape of Good Hope ($34^{\circ} 21' \text{ S.}$, $18^{\circ} 29' \text{ E.}$)

Latitude:	40° N.	20° N.	0°	20° S.
Longitude:	$42^{\circ} 15' \text{ W.}$	$22^{\circ} 25' \text{ W.}$	$8^{\circ} 27' \text{ W.}$	$5^{\circ} 31' \text{ E.}$

3. Honolulu ($21^{\circ} 18' \text{ N.}$, $157^{\circ} 52' \text{ W.}$) to Vancouver ($48^{\circ} 34' \text{ N.}$, $124^{\circ} 08' \text{ W.}$)

Latitude:	$30^{\circ} 24' \text{ N.}$	$35^{\circ} 12' \text{ N.}$	$39^{\circ} 19' \text{ N.}$	$42^{\circ} 48' \text{ N.}$
Longitude:	150° W.	145° W.	140° W.	135° W.

4. Cape Farewell ($59^{\circ} 47' \text{ N.}$, $43^{\circ} 40' \text{ W.}$) to Bouvet Island ($54^{\circ} 28' \text{ S.}$, $3^{\circ} 30' \text{ E.}$)

Latitude:

50° N.	30° N.	10° N.	10° S.	30° S.	40° S.
-------------------------	-------------------------	-------------------------	-------------------------	-------------------------	-------------------------

Longitude:

$35^{\circ} 20' \text{ W.}$	$26^{\circ} 04' \text{ W.}$	$20^{\circ} 08' \text{ W.}$	$14^{\circ} 58' \text{ W.}$	$9^{\circ} 02' \text{ W.}$	$0^{\circ} 14' \text{ E.}$
-----------------------------	-----------------------------	-----------------------------	-----------------------------	----------------------------	----------------------------

The Types of Navigation. The problems of determining the direction and distance of one point from another and of fixing the position of an airplane or a ship are solved in a variety of ways. Position

may be determined by the aid of maps and visible landmarks; this is called *piloting*. Position is also found by *dead reckoning*; that is, calculating the new position from the distance and direction flown or sailed from some given position. Radio has become increasingly important in navigation. The *radio beam* and the *radio direction finder* tell the direction of the airplane from the radio station and enable the navigator to plot these direction lines on a map. Finding latitude and longitude from observations of the sun and stars is called *celestial navigation*. Whenever possible these four methods of navigation are used together as is shown in this description of the training at the Kelly Field Navigation School.

Actually the aerial navigator has to learn four types of navigation and practice all of them in the one hundred hours in the air he gets here besides the classroom work. In one of AT-7 Beechcraft navigation-training planes used here you will find three student navigators all working at once, but separately, while the ship is speeding on one of the training missions. These average one thousand miles apiece, and take the boys to every part of this country within a 500-mile radius.

One man will be piloting, with the aid of maps, landmarks, beacons and some of the plane's instruments. Another will be working on dead reckoning, using compass, drift indicator, clock, and other instruments. The third will be navigating wholly by radio, correlating radio beacons, beams, and other sources of information obtainable by radio.

Or in place of one of these three, there being room for only three students in an AT-7, there will be one man practicing celestial navigation, taking sights with a bubble sextant through the plane's top hatch or windows of the sun, moon or stars, and working them out with his own tables and instruments. If all three men come out with the same answer, and the plane is where they think it is when they say it should be, the mission has been a success.

— WILLIAM H. TAYLOR in the *New York Herald Tribune*,
July 26, 1942

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CHAPTER II

THE SAILINGS AND DEAD RECKONING

Dead reckoning is the process of finding the new position of a ship or plane from the distance and direction it has traveled from some given position. The various methods used in dead reckoning and in the reverse problem of finding the direction and distance between two points are called *sailings*.

Measurement of Directions or Bearings. In problems of navigation it is necessary to have an exact method of indicating directions. The direction of point A from point B is called the *bearing* of A from B and is expressed as an angle. Bearings are indicated from 0° to 360° ; north is 0° , east 90° , south 180° , and west 270° , as shown in Figure 13.

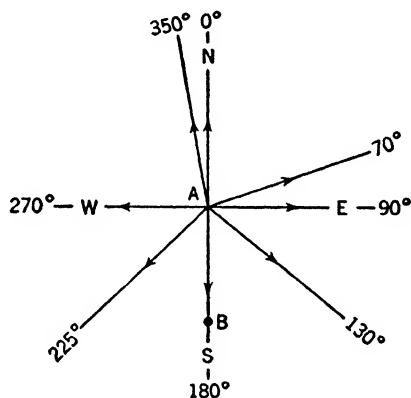


FIGURE 13.

If point B is due south of point A, then the bearing of B from A is 180° , and a plane flying from A to B would be on a *course* of 180° . If a plane follows a course of 130° , then it is flying in a direction which is 40° toward the south from east. A northeast course or bearing is 45° , southeast 135° , southwest 225° , and northwest 315° . This method of indicating bearings and courses has largely replaced the older method of dividing the 360° into the 32 points of the compass.

The most common instrument for measuring bearings is the magnetic compass. The compass needle does not usually point to the true north and hence a correction must be applied for this difference. The difference between true north and magnetic north is called *magnetic variation*, or declination, and may be east or west. The values of the magnetic variation for a few places in the United States for 1942 are given on page 16.

Topeka	9° 13' E.	Bismarck	13° 47' E.
Louisville	0° 40' E.	Houston	9° 07' E.
Annapolis	7° 42' W.	Norfolk	6° 19' W.
Buffalo	7° 34' W.	Cheyenne	14° 44' E.

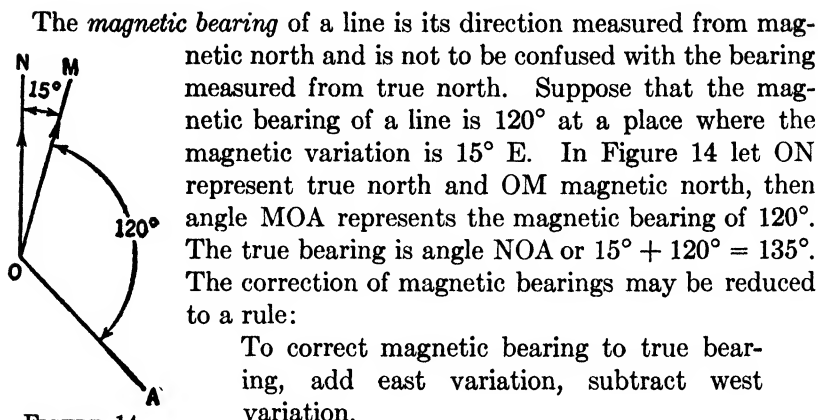


FIGURE 14.

To correct magnetic bearing to true bearing, add east variation, subtract west variation.

This rule should not be applied arbitrarily until it has been verified by many examples.

There are other errors of the compass needle caused by local magnetism and by the magnetism of an aircraft. Such errors must be determined and applied in the same way as variation.

Problems

1. Construct a figure like Figure 13 to show bearings or courses of 320°, 190°, 10°, 97°, 210°, 175°, 290°.
2. Find your local magnetic variation by reference to a table or large-scale map.
3. Complete the following table. In each case construct a figure like Figure 14. (Do not write in your text.)

Magnetic Bearing	Variation	True Bearing
73°	5° E.	-----
310°	10° E.	-----
230°	15° W.	-----
90°	4° E.	-----
175°	9° W.	-----
260°	10° W.	-----
20°	8° E.	-----
130°	6° W.	-----

4. Complete the following table.

<i>Magnetic Bearing</i>	<i>Variation</i>	<i>True Bearing</i>
-----	6° W.	30°
-----	11° W.	63°
-----	14° E.	330°
-----	5° W.	250°
-----	10° E.	185°
-----	15° W.	80°
-----	8° E.	240°

Plane Sailing. When it is required to find merely the relation between the course, distance sailed, and the corresponding north-south and east-west distances, a method known as plane sailing is used. Plane sailing is concerned with a triangle like ABC, in Figure 15, where angle C is the course from C to A, CB is the meridian through C, and BA the parallel through A. Plane trigonometry may be used to determine the various parts of triangle ABC.

CA is called the distance sailed (Dist.).

CB the difference of latitude (DL).

BA is called the departure (Dep.).

Notice that the length of CB in nautical miles may be found directly from the difference of the two latitudes; if A is in latitude 47° and C in latitude 43°, then CB = 4° or 240 nautical miles. BA, however, is an arc of a parallel and its length in miles would *not* be equal to the difference of longitude. One minute on the equator is one nautical mile, but one minute on the other parallels will vary according to the latitude. Hence difference of latitude (DL) may refer either to nautical miles or difference of minutes of latitude, but departure refers only to a distance expressed in nautical miles. From this it follows that plane sailing may not be used to find the longitude of a new position.

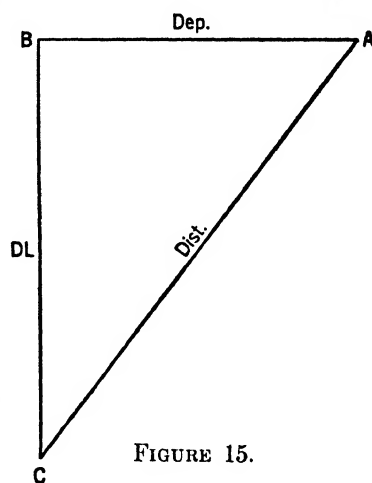


FIGURE 15.

The two problems of plane sailing are:

- (1) Given the course and distance, to find the difference of latitude and departure.
- (2) Given the difference of latitude and departure, to find the course and distance.

The formulas for plane sailing are simply the formulas for solving a plane right triangle.

$$\begin{aligned} DL &= \text{Dist.} \cos C \\ \text{Dep.} &= \text{Dist.} \sin C \\ \tan C &= \frac{\text{Dep.}}{DL} \end{aligned}$$

When the third formula is used to find angle C , an acute angle will be found in the tables; this acute angle will be the angle from the meridian to the distance. The relationship between angle C and the course, expressed in the usual 0° to 360° manner, is shown in Figure 16. In plane sailing as in most problems of trigonometry a figure should always accompany the solution of a problem. Draw figures for examples 1 and 2.

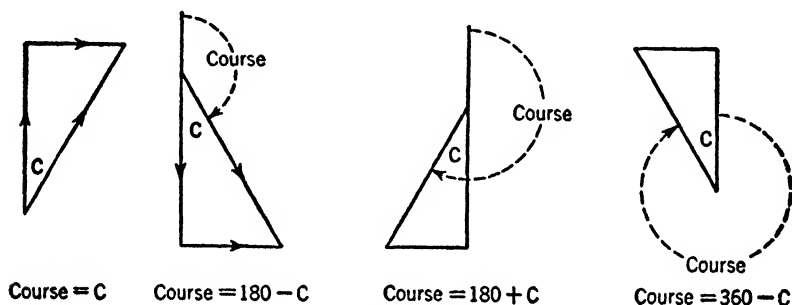


FIGURE 16.

Plane sailing may not be used to find the difference of longitude between C and A , and may not be used when the distances involved become too long, because of the neglect of curvature.

Example 1. A plane flies a course of 335° from Mitchel Field ($40^\circ 45' \text{ N.}$, $73^\circ 58' \text{ W.}$) for 200 nautical miles. Find the difference of latitude and departure.

$\log 200 = 2.30103$	$\log 200 = 2.30103$
$\log \cos 25^\circ = 9.95728$	$\log \sin 25^\circ = 9.62595$
$\log DL = 2.25831$	$\log \text{Dep.} = 1.92698$
$DL = 181.3 \text{ nautical miles}$	$\text{Dep.} = 84.52 \text{ nautical miles}$

Example 2. Find the course and distance from C to A if A is 215 nautical miles west and 183 nautical miles south of C .

$\log 215 = 2.33244$	$\log 215 = 2.33244$
$\log 183 = 2.26245$	$\log \sin 49^\circ 36' = 9.88169$
$\log \tan C = .06999$	$\log \text{Dist.} = 2.45075$
$C = 49^\circ 36'$	$\text{Dist.} = 282.3 \text{ nautical miles}$
$\text{Course} = 229^\circ 36'$	

Problems

1. A ship sails a course of 140° for a distance of 283 miles. Find the difference of latitude and departure.

2. A ship changes her latitude 207 miles to the north and makes a departure of 118 miles to the east. What course has she followed, and what is the distance made good?

3. A plane follows a course of 310° for $2\frac{1}{2}$ hours at 100 miles per hour. Find the difference of latitude and departure.

4. Find the course and distance in the following:

<i>DL</i>	<i>Dep.</i>
(a) 250 miles north	300 miles east
(b) 298 miles south	267 miles west
(c) 315 miles south	245 miles east
(d) 135 miles north	210 miles west

5. Find the difference of latitude and departure in the following:

<i>Course</i>	<i>Distance</i>
(a) $37^\circ 34'$	384 miles
(b) $307^\circ 10'$	242 miles
(c) $145^\circ 00'$	317 miles
(d) $210^\circ 40'$	289 miles
(e) $298^\circ 30'$	117 miles

Parallel Sailing. If the course is due east or west, it is easy to convert the nautical miles of departure into degrees and minutes of longitude, or to convert a given difference of longitude into nautical miles. In Figure 17, AB is an arc of the parallel of L° , DE the corresponding arc of the equator, O the center of the earth, and O'A the radius of the parallel. In right triangle OAO',

$$O'A = OA \cos L = OD \cos L$$

$$\frac{AB}{DE} = \frac{O'A}{OD} = \frac{OD \cos L}{OD}$$

Hence

$$AB = DE \cos L$$

AB is the departure or the distance along the parallel, and DE is the corresponding distance along the equator. The number of nautical miles in DE equals the difference of longitude, since on the equator one minute of longitude equals one nautical mile. Differ-

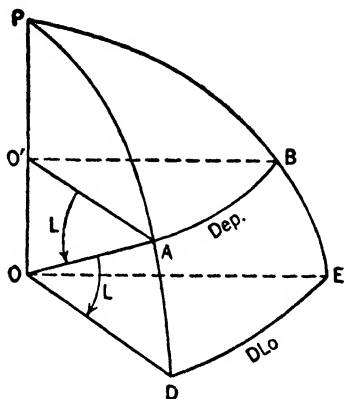


FIGURE 17.

ence of longitude is abbreviated DLo. For parallel sailing the formulas are:

$$\text{Dep.} = \text{DLo} \cos L, \text{ and } \text{DLo} = \frac{\text{Dep.}}{\cos L}$$

Notice that the difference of longitude, DLo, is the number of degrees and minutes of longitude between the two meridians and is not always the differences between the two longitudes. For the three examples below $\text{DLo} = 10^\circ = 600'$.

$$\begin{array}{lll} 60^\circ \text{ W. to } 70^\circ \text{ W.} & & 70^\circ - 60^\circ = 10^\circ \\ 5^\circ \text{ E. to } 5^\circ \text{ W.} & & 5^\circ + 5^\circ = 10^\circ \\ 175^\circ \text{ W. to } 175^\circ \text{ E.} & 360 - (175^\circ + 175^\circ) = 10^\circ \end{array}$$

Example 1. Find the distance along the parallel of $37^\circ 30' \text{ S.}$ from White Island, New Zealand ($177^\circ 11' \text{ E.}$), to Cape Howe, Victoria ($149^\circ 59' \text{ E.}$).

$$\begin{aligned} \text{DLo} &= 177^\circ 11' - 149^\circ 59' = 27^\circ 12' = 1632' \\ \text{Dep.} &= 1632 \cos 37^\circ 30' = 1295 \text{ nautical miles} \end{aligned}$$

Example 2. A ship sails east from ($32^\circ 00' \text{ S.}, 2^\circ 10' \text{ W.}$) for 300 nautical miles. Find the longitude of the new position.

$$\begin{aligned} \text{DLo} &= 300 / \cos 32^\circ = 354' = 5^\circ 54' \\ \text{New longitude} &= 2^\circ 10' \text{ W.} - 5^\circ 54' = 3^\circ 44' \text{ E.} \end{aligned}$$

Problems

1. Complete the following table. Find the DLo in degrees and minutes.

$43^\circ 10' \text{ W.}$	$54^\circ 48' \text{ W.}$
$178^\circ 13' \text{ W.}$	$176^\circ 19' \text{ E.}$
$6^\circ 17' \text{ E.}$	$5^\circ 46' \text{ W.}$
$74^\circ 00' \text{ W.}$	$81^\circ 19' \text{ W.}$
$120^\circ 47' \text{ E.}$	$122^\circ 27' \text{ E.}$
$174^\circ 39' \text{ E.}$	$169^\circ 43' \text{ W.}$

2. Find the number of nautical miles in 10° of longitude on each of the parallels of $15^\circ, 30^\circ, 45^\circ, 60^\circ, 75^\circ, 90^\circ$.

3. Find the number of minutes of longitude in 600 nautical miles on each of the parallels of $15^\circ, 30^\circ, 45^\circ, 60^\circ, 75^\circ$.

4. Find the distance along the parallel of $35^\circ 39' \text{ N.}$ from Tokyo ($139^\circ 45' \text{ E.}$) to Wang-kia-tia Bay, China ($119^\circ 52' \text{ E.}$).

5. A ship in latitude $58^\circ 00' \text{ N.}$ and longitude $23^\circ 45' \text{ W.}$ sails west 195 nautical miles. Find the longitude of the new position.

6. Find the distance along the parallel of $41^\circ 09' \text{ N.}$ from Bridgeport Harbor ($73^\circ 11' \text{ E.}$) to the north point of Gardiners Island ($72^\circ 09' \text{ W.}$).

7. A ship in latitude $32^{\circ} 22' N.$ and longitude $52^{\circ} 20' W.$ sails west 365 miles. Required, her distance from the Island of Bermuda in the same latitude and longitude $64^{\circ} 43' W.$ (from Norie's *Navigation*).

8. Find the distance along the parallel of $41^{\circ} 09' N.$ from Oporto Light ($8^{\circ} 41' W.$) to Block Island Light ($71^{\circ} 33' W.$).

Middle Latitude Sailing. When the course between two points does not follow a parallel of latitude, some method must be found to obtain the longitude of the new position. In parallel sailing, difference of longitude was found by the formula $DLo = Dep./\cos L.$ In middle latitude sailing, nautical miles of departure are converted to minutes of longitude by dividing by the cosine of the average of the two latitudes, that is, the middle latitude. Middle latitude is abbreviated $L_m.$

Middle latitude sailing is equivalent to solving a plane right triangle ABC, Figure 18, where the sides are DL, $DLo \cos L_m,$ and the distance. Angle C is related to the course as was shown in Figure 16, page 18.

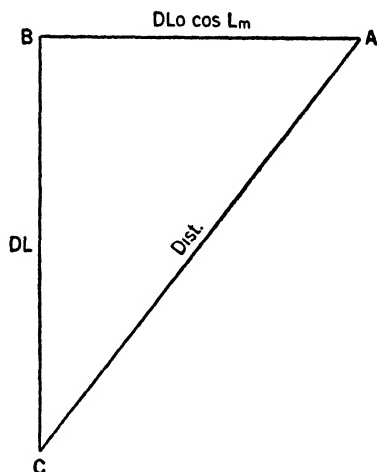


FIGURE 18.

The formulas for finding course and distance are:

$$\tan C = \frac{Dep.}{DL} = \frac{DLo \cos L_m}{DL}, \text{ and } Dist. = \frac{Dep.}{\sin C}$$

Example 1. Find the course and distance by middle latitude sailing from ($35^{\circ} 00' N., 61^{\circ} 31' W.$) to ($30^{\circ} 00' N., 53^{\circ} 22' W.$).

$$DL = 35^{\circ} 00' - 30^{\circ} 00' = 5^{\circ} 00' = 300' = 300 \text{ nautical miles}$$

$$DLo = 61^{\circ} 31' - 53^{\circ} 22' = 8^{\circ} 09' = 489'$$

$$L_m = \frac{35^{\circ} 00' + 30^{\circ} 00'}{2} = 32^{\circ} 30'$$

$$\log DLo(489) = 2.68931$$

$$\log Dep. = 2.61534$$

$$\log \cos L_m(32^{\circ} 30') = 9.92603$$

$$\log \sin C(53^{\circ} 58') = 9.90777$$

$$\log Dep. = 2.61534$$

$$\log Dist. = 2.70757$$

$$\log DL(300) = 2.47712$$

$$Dist. = 510.0 \text{ nautical miles}$$

$$\log \tan C = 0.13822$$

$$C = 53^{\circ} 58'$$

$$\text{Course} = 126^{\circ} 02'$$

When the course and distance are given, the formulas for finding the latitude and longitude of the new position are as follows:

$$DL = \text{Dist. cos. } C \qquad DLo = \frac{\text{Dist. sin } C}{\cos L_m}$$

The difference of latitude and longitude are then applied to the given latitude and longitude to find the new position.

Example 2. A plane leaves Dayton Airport ($39^\circ 53' \text{ N.}$, $84^\circ 12' \text{ W.}$) and flies a course of 225° for 400 miles. Find the latitude and longitude of the new position.

$\log \text{Dist.}(400) = 2.60206$ $\log \cos C(45^\circ) = \underline{9.84949}$ $\log DL = 2.45155$ $DL \text{ (nautical miles)} = 282.8$ $DL = 4^\circ 42'.8$ $L_m = 39^\circ 53' - \frac{4^\circ 42'.8}{2}$ $L_m = 37^\circ 32'$	$\log \text{Dist.}(400) = 2.60206$ $\log \sin C(45^\circ) = \underline{9.84949}$ $\log \text{Dep.} = 2.45155$ $\log \cos L_m(37^\circ 32') = \underline{9.98927}$ $\log DLo = 2.55228$ $DLo = 356'.6$ $DLo = 5^\circ 57'$
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$$\begin{aligned} \text{New position: } 39^\circ 53' \text{ N.} - 4^\circ 43' &= 35^\circ 10' \text{ N.} \\ 84^\circ 12' \text{ W.} + 5^\circ 57' &= 90^\circ 09' \text{ W.} \end{aligned}$$

Middle latitude sailing is an approximate method and cannot be used when the distance is too great. The error is considered to be "immaterial for distances ranging up to 200 nautical miles, but for distances of 600 nautical miles the default from accuracy would be of the order of one per cent when the middle latitude is as great as 60° " (Bowditch). Middle latitude cannot be used for courses crossing the equator.

Problems

1. Find the course and distance between the following points on the great circle from New York to the Cape of Good Hope.

- A. ($25^\circ 00' \text{ N.}$, $46^\circ 36' \text{ W.}$)
- B. ($20^\circ 00' \text{ N.}$, $40^\circ 41' \text{ W.}$)
- C. ($15^\circ 00' \text{ N.}$, $35^\circ 20' \text{ W.}$)
- D. ($10^\circ 00' \text{ N.}$, $30^\circ 13' \text{ W.}$)

2. Find the new position from the following initial positions and courses and distances.

- A. ($49^\circ 42' \text{ N.}$, $50^\circ 00' \text{ W.}$), course $49^\circ 20'$, distance 248.6 nautical miles
- B. ($52^\circ 24' \text{ N.}$, $45^\circ 00' \text{ W.}$), course $53^\circ 18'$, distance 222.5 nautical miles
- C. ($54^\circ 37' \text{ N.}$, $40^\circ 00' \text{ W.}$), course $56^\circ 42'$, distance 210.6 nautical miles

3. A ship sails 268 nautical miles on a course of 223° from latitude $39^\circ 37' \text{ S.}$ and longitude $3^\circ 42' \text{ E.}$ Find the new position.

4. Find the course and distance by middle latitude sailing from Vladivostok ($43^{\circ} 05' N.$, $131^{\circ} 53' E.$) to Niigati ($37^{\circ} 55' N.$, $139^{\circ} 03' E.$).
5. Find the position reached by a ship sailing a course of 315° for 300 nautical miles from ($48^{\circ} 46' N.$, $178^{\circ} 15' W.$).

Bibliography

World Almanac, 1942: "Table of Magnetic Variation," 175.

Magnetic variation is indicated on the charts published by the U. S. Hydrographic Office. These charts cover nearly every part of the world; inquiries to the Hydrographic Office should state whether a general chart of the entire region is desired or whether the writer wishes the more detailed, larger-scale charts covering it. Magnetic variation is also indicated on the "Sectional and Regional Aeronautical Charts of the United States" sold by the U. S. Coast and Geodetic Survey.

The Magnetic Compass: Dutton, Chapter III; H. O. 216, 39-56; Bowditch, Chapter III.

Plane, Parallel and Middle Latitude Sailing: Dutton, 83-91; Bowditch, 98-103.

Mercator Sailing: Dutton, 91-93; Bowditch, 103-105.

Air Navigation by Dead Reckoning: Lyon, Chapter V.

CHAPTER III

PILOTING

Navigation beyond the sight of land, where observations of the sun and stars are necessary for the determination of latitude and longitude, is called *celestial navigation*. Celestial navigation is to be distinguished from navigation along coasts and in harbors which is termed *piloting*. "Piloting, in the sense given to the word by modern and popular usage, is the art of conducting a vessel in channels and harbors and along coasts, where landmarks and aids to navigation are available for finding the position, and where the depth of water and dangers to navigation are such as to require a constant watch to be kept upon the vessel's course and frequent changes to be made therein" (Bowditch). The greatest dangers to ships usually occur when the ship is near land or is entering or leaving a harbor; and while errors in finding position on the high seas may be corrected by later observations, an error in piloting is frequently disastrous.

In modern piloting the navigator is provided with detailed charts of the coasts, lists of landmarks such as lights and buoys, and precise information concerning the tides, currents, and depth of the water. Soundings are of the utmost importance in piloting. Modern instruments for taking soundings measure the time interval required for a sound wave transmitted from the bottom of the vessel to be echoed back to the vessel. Since sound travels at a constant speed in water, the depth of the water can be automatically determined and plotted by the sounding machine. When the position of an aircraft, or rather the point on the earth directly beneath it, is determined from visible landmarks, methods corresponding to those of piloting may be used. Air navigation, however, is complicated by the rapid change of position and by frequent poor visibility. The aeronautical charts of the United States issued by the U. S. Coast and Geodetic Survey provide the navigator of a plane with detail similar to that of the coast and harbor charts.

Position by Cross Bearings. When it is possible to observe the bearings of two, or more, objects which are indicated on the navigational or aeronautical chart, position is easily found by plotting these bearing lines. The navigator observes the bearing of a lighthouse

from his ship, but must plot on the map the bearing of the ship from the lighthouse. In Figure 19 the bearing of point B measured from A is 110° , but the bearing of A from B is $110^\circ + 180^\circ$ or 290° ; similarly the bearing of D measured from C is 240° , but the bearing of C from D is 60° , or $240^\circ - 180^\circ$.

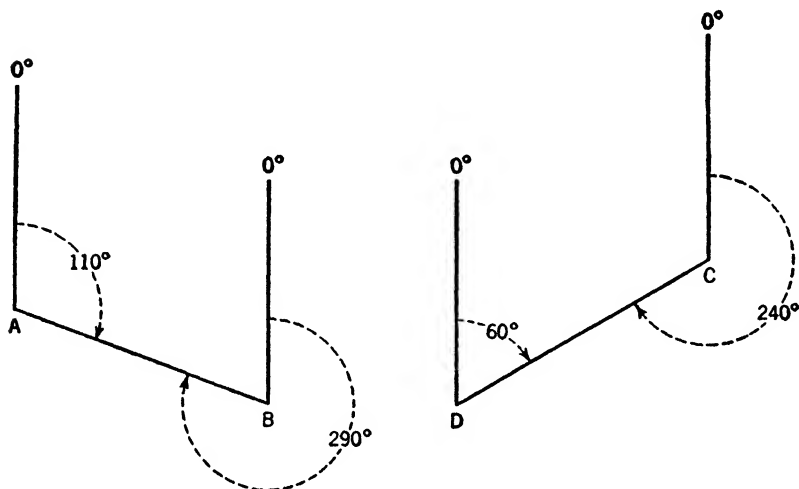


FIGURE 19.

In Figure 19 if A represents a ship and B a lighthouse, then the navigator measures the bearing of 110° , but must plot the *reciprocal* bearing of 290° on his chart. Reciprocal bearings differ by 180° . In problems of this kind the meridians through A and B are considered to be parallel straight lines. Two bearing lines from different objects will intersect in a point, but give no check on the accuracy of the intersection. When possible it is desirable to obtain three bearings and have a check on the accuracy of the intersection, or *fix*.

Example. A plane flying over Long Island Sound observes the bearings of three lighthouses.

Plum Island, 275°
 Gardiners Island, 265°
 Montauk Point, 140°

Find the position of the plane on the chart.

The plane bears

95° from Plum Island
 85° from Gardiners Island
 320° from Montauk Point

The three bearings do not determine a point but a small triangle, the so-called "cocked hat," or triangle of error. The accuracy of the fix depends of course on the size of this triangle.

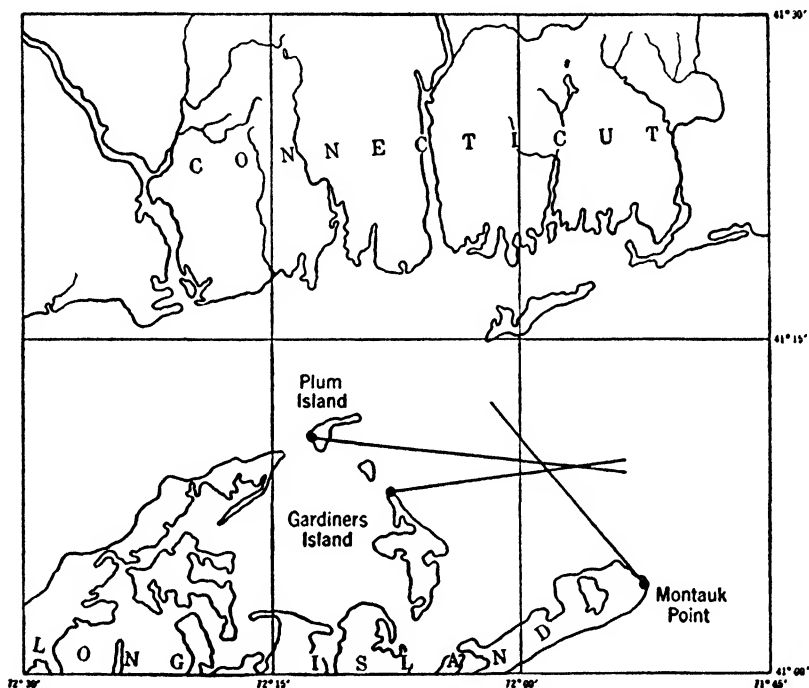


FIGURE 20.

Problems

1. Find the reciprocal bearings for the following bearings. Construct in each case a figure like those of Figure 19. (Do not write in your text.)

<i>Bearing of A from B</i>	<i>Bearing of B from A</i>
350°	-----
175°	-----
87°	-----
247°	-----
190°	-----
10°	-----
298°	-----
90°	-----
37°	-----

2. Construct a triangle ABC with $AB = 3.5$ miles, $BC = 4.0$ miles, and $CA = 5.4$ miles. Make the direction of AB, 220° . AC is east of B. A navigator observes these bearings from his ship; A, 295° ; B, 270° ; C, 250° . Find the position of his ship on your figure.

3. AB is 3.8 miles long and has a direction of 130° , BC is 2.8 miles, and CA 5.6 miles. AC is west of B. A navigator observes the bearing of A to be 35° , B 60° , and C 80° . Find the position of his ship.

Radio Navigation. Various radio devices have been developed which are of the greatest importance in modern air and marine navigation. There are three means of finding position by radio: the radio beam, bearings of the plane or ship from the radio station, and bearings of the radio stations from the plane or ship. These methods are used to determine position in much the same way as cross bearings.

Along the main airlines in the United States there are stations at frequent intervals which broadcast signals for the guidance of pilots. These signals consist simply of the letters A ($\cdot -$) and N ($- \cdot$) in Morse code sent out in opposite directions, so that each letter is audible in two opposite quadrants or sectors. Along

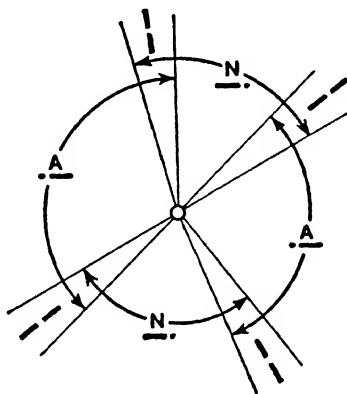


FIGURE 21.

the boundaries of adjacent sectors the two signals merge in a continuous signal ($--$); these equisignal zones are the *radio beams*. A pilot approaching the transmitting station will receive one letter more pronounced if he is to the left of the beam and the other more pronounced if he is to the right of the beam. The letters A and N are interrupted frequently for other signals which identify the direction of the beam.

The *radio direction finder* is an instrument which determines the direction from which a radio message is received. By means of the radio direction finder a station may determine the bearing of a plane from the station and then transmit this bearing back to the plane. Two stations can give the plane intersecting bearings which determine the position of the plane. In preparation for their trans-Pacific flights the engineers of Pan American Airways developed radio direction finders with a range of two thousand miles. Radio direction

finders have also been developed for the plane to carry, so that it may find its own bearing from a ship or land station. Radio direction finders of this type have been developed with a range of one thousand miles.

Radio bearing lines may be plotted on a chart and used to find a fix in much the same way as the bearings obtained from visible objects. Radio waves follow a great circle, and for long distances the great circle will not have the same direction as the rhumb line (see Figure 12). Before a radio bearing is plotted as a straight line on a Mercator chart, the bearing must be corrected by special tables.

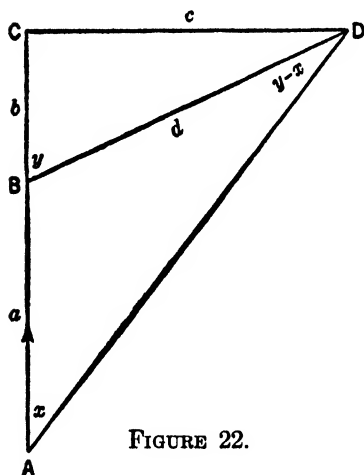


FIGURE 22.

Position by Two Bearings of a Single Object. The position of a ship may be determined by two bearings of a single object obtained at different times. In Figure 22, A and B are two positions of a ship following the course AB, and D is the object sighted. The angles x and y , called *angles on the bow*, are obtained from the bearings of

D, measured at A and B. AB is known, since it is the distance run between the times of the two observations. Figure 22 is simply a triangle ABD with altitude c , and plane trigonometry may be used to find d , the distance from the object at the time the second bearing is taken, or to find c the perpendicular. The distance CD (c) is called the *distance at which the object is passed abeam*. In practice the use of trigonometry is avoided by special tables.

There are numerous special cases of Figure 22. If angle $y = 2x$, then $BD = AB$, or the distance run equals the distance to the object at the second observation. This is called *doubling the angle on the bow*. In this case any convenient value of x may be taken and the angle observed until it has doubled.

There is a special method of finding DC, the distance abeam. This consists of selecting the two angles x and y so that $\cot x - \cot y = 1$. For example, if $x = 30^\circ$, then $\cot x = 1.7321$ and $\cot y = .7321$, using a table $y = 53^\circ 48'$. The navigator is supplied with special tables which give corresponding values of x and y for this special case.

Problems

1. Prove that $d = \frac{a \sin x}{\sin (y-x)}$ and that $c = \frac{a \sin x \sin y}{\sin (y-x)}$.

2. Prove that $BD = AB$ if $y = 2x$.

3. Prove that if $\cot x - \cot y = 1$, then $a = c$.

4. Prove that $\frac{\sin x \sin y}{\sin (y-x)} = \frac{1}{\cot x - \cot y}$.

5. Find values of x and y , such that $\cot x - \cot y = 1$ by completing the following table. Find the other angles to the nearest degree only. Hint: Unless you have access to a table of natural trigonometric functions, you will have to use both the logarithms of numbers (pages ii-xix) and the log cotangents (pages xx-xlii) to solve this problem.

<i>Cot x</i>	<i>x</i>	<i>y</i>	<i>Cot y</i>
2.475	22°	----	----
----	----	41°	1.150
1.963	27°	----	----
----	----	51°	0.810
1.600	32°	----	----
----	----	79°	0.194

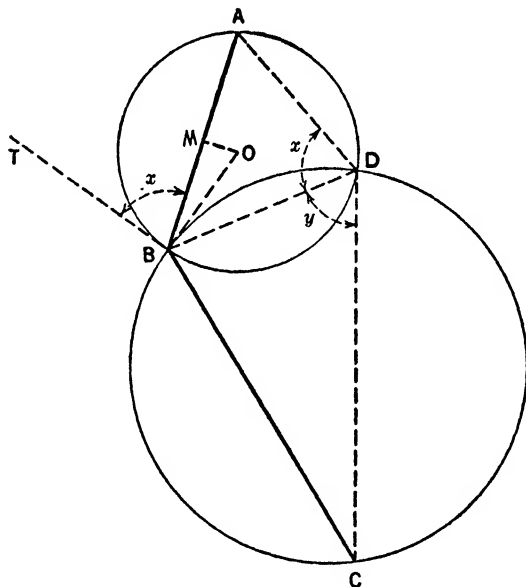


FIGURE 23.

The Three-Point Problem. An important method of fixing position is to measure the angles which the ship makes with three known points. In Figure 23, let A, B, and C be three visible points and D

the ship. D may be determined on the chart by the angles x and y . Since the solution depends only on x and y and not on bearings, the angles may be measured with a sextant and compass errors eliminated. Errors caused by estimated distances are also eliminated.

The simplest solution consists merely in laying out the angles on a sheet of tracing paper and moving the paper over the chart until the sides of the angles pass through A , B , and C . D may be located in a similar manner by using a three-arm protractor. Since D lies on the intersection of two circles which have AB and BC as chords, the problem may also be solved by a construction of geometry: "To construct, on a line segment $[AB]$ as a chord, a segment of a circle in which a given angle $[x]$ may be inscribed." If this construction is repeated with the line BC and angle y , D is found by the intersection of the two circles.

To determine O , the center of the circle through A and B , construct:

angle $ABT = \text{angle } x$

BO perpendicular to BT

MO the perpendicular bisector of AB

The position of D is indeterminate from the given data when the two circles coincide; and for an accurate fix it is necessary to avoid the case where the centers are close together. In general, the position of D is determined more accurately when the angles x and y are large and the distances from D to A , B , and C are short.

The three-point problem was first used by the Dutch mathematician, Willebrord Snell, who published a trigonometrical solution in 1617. It has many applications in marine surveying and military engineering as well as in navigation. Many solutions other than the geometrical one have been devised.

Problems

1. Use the construction of the three-point problem to locate a ship D from three lights, A , B , and C . $AB = 2.1$ miles and bears due south, $BC = 3.2$ miles and bears 164° . Angle $ADB = 33^\circ$, angle $BDC = 68^\circ$. The ship is on the east side of AB .
2. Give the proof for the geometric construction in Figure 23.
3. Construct the circle through AB in Figure 23 by making an isosceles triangle on AB as a base. Make each base angle of the isosceles triangle equal to $90^\circ - x/2$.
4. Construct the circle through AB in Figure 23 by constructing a triangle on AB as a base with the sum of the base angles equal to $180^\circ - x$.

The Danger Angle. Another application of elementary geometry is used in sailing along a coast where there are hidden reefs or other dangers. In Figure 24, A and B are two lighthouses and C a hidden danger to be avoided. The navigator constructs a circle on the chart which passes through A and B and safely includes C. He then measures the angle ADB inscribed in the larger arc AB. In order to avoid C, the ship must be steered so that the angle ASB, which it makes with A and B, is less than angle ADB.

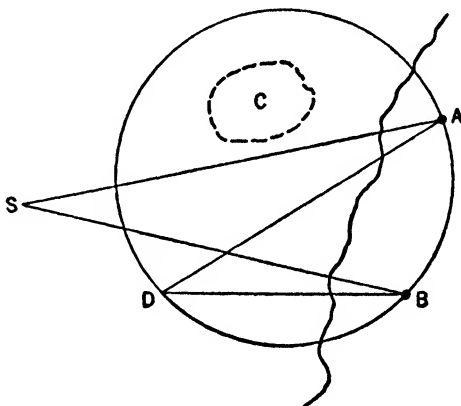


FIGURE 24.

Problems

1. Prove that (a) the ship is inside the circle when angle S is greater than angle D; (b) the ship is on the circle when angle S equals angle D; (c) the ship is outside the circle when S is less than angle D.
2. Construct a figure to show how a double danger angle might be used to steer a safe course between two dangers.

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CHAPTER IV

SPECIAL PROBLEMS OF AIR NAVIGATION

The same basic problems occur in both air and sea navigation. Courses and distances must be calculated for both planes and ships, and frequent determination of position must be made to see if the plane or vessel is on course. The aerial navigator must work more rapidly and frequently with more uncertain data. Ocean currents have been charted so that their effect on the course of a vessel can be accurately determined. Although it is impossible to chart the winds in the same way as the ocean currents, the navigator of a plane must make due allowance for the speed and direction of the wind in determining his course. The actual direction in which a plane goes, called its *track*, is the result of the wind direction and the direction in which the plane is aimed or pointed. Similarly the speed of a plane is the result of the wind velocity and the speed which the plane has in calm air.

The high speed of modern aircraft makes it imperative that computations of course and distance be made rapidly. If the position of a ship is in doubt, the vessel may "heave to" until an accurate fix has been obtained. The aerial navigator must make the best determination possible while traveling at a relatively high speed. Many of the special problems of air navigation are solved by graphic construction, since the solutions must be obtained quickly, and since the given data are frequently approximate. This chapter will present solutions of certain problems peculiar to air navigation.

The Triangle of Velocities. An important problem in air navigation is to determine the effect of a wind of known velocity and direction on the course and speed of a plane. Consider a northwest wind of forty miles an hour and a plane capable of making 150 miles an hour in calm air. Suppose the plane starts from O, Figure 25, in a northeast direction — that is, on a course of 45° . C in Figure 25 represents the position of the plane after one hour if there is no wind; OC has a bearing of 45° and a length representing 150 miles to scale. The effect of the wind alone would be to move the plane in a southeast direction. OA with a bearing of 135° and a length of 40 miles to the same scale as AC represents the effect of the wind, that is,

the wind itself in one hour would move the plane from O to A. The plane of course does not follow either OA or OC, but its actual direction lies between OA and OC.

To determine the actual course and speed of the plane, construct the parallelogram OABC and draw the diagonal OB. The plane actually moves along the line OB, with a speed represented by the length of OB, and a course represented by the direction of OB. For this example, the effective speed of the plane is 155 miles an hour and its actual course is 60° .

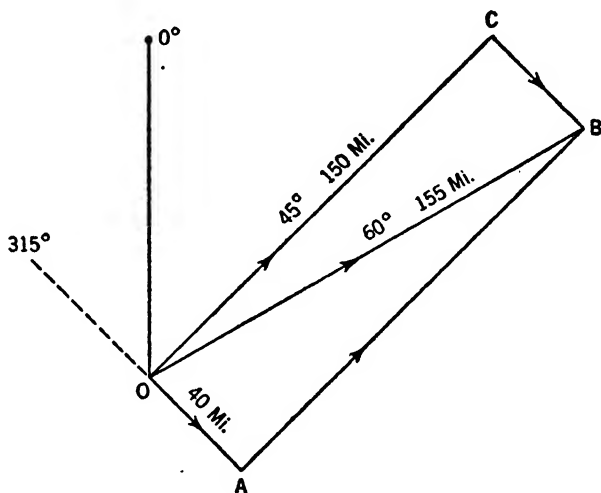


FIGURE 25.

The above example shows that it is necessary to distinguish carefully between the direction in which the plane is pointed and the direction in which it actually goes. *Heading* is the direction in which a plane is pointed; in Figure 25 the heading is 45° . *Track* is the direction of travel over the ground, 60° in Figure 25. The *air speed* or the speed which the plane has in calm air must also be carefully distinguished from the speed over the ground or *ground speed*. In Figure 25 the air speed is 150 miles, measured on OC, and the ground speed is 155 miles, measured along OB.

The *drift angle* is the angle between the heading and the track. In Figure 25 the drift is angle COB, 15° to the right. Drift angle, track, and ground speed may be determined by drawing only triangle OAB instead of the entire parallelogram.

In Figure 25 a heading was assumed and the corresponding track

was found. In general the navigator has to find the heading which corresponds to a given track. This problem is also solved by constructing a triangle or parallelogram of velocities. Suppose it is required to find the heading for a track of 80° with the same wind and air speed as before. In Figure 26, construct OA with a bearing of 135° and a length of 40 miles to represent the wind. Draw the indefinite line OB with a bearing of 80° to represent the given track; then with center A, and a radius of 150 miles, draw an arc intersecting OB at B. Complete the parallelogram OABC. As before, head-

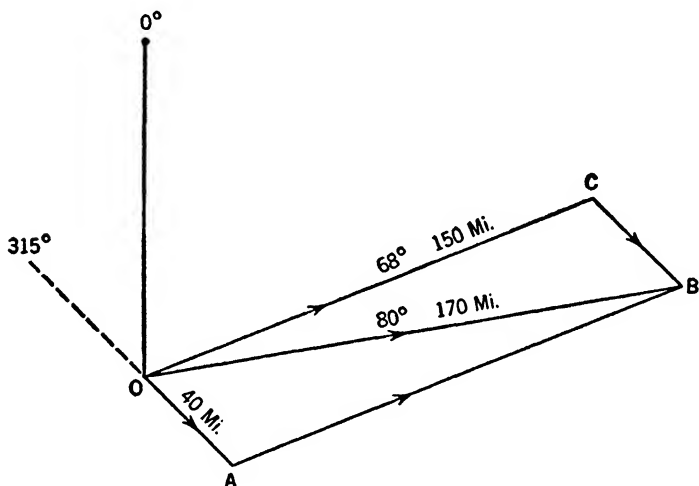


FIGURE 26.

ing and air speed are represented by OC, and track and ground speed by OB.

OC represents the given air speed, 150 mi./hr., and the required heading, 68° .

OB represents the required ground speed, 170 mi./hr., and the given track, 80° .

The drift angle is 12° to the right, angle COB.

In Figure 26, the pilot must steer the plane 12° to the left of the desired track, in order to offset the force of the wind driving him off his course. That is, the pilot steers in the direction OC, in order for the plane to go in the direction OB.

In aerial navigation, heading is determined from the compass, and air speed from the air-speed indicator. Track and ground speed must be found by some method corresponding to the above. The

wind direction is indicated in the usual manner, from 0° to 360° ; however, the wind is said to be *from some given direction*; a north wind is a *wind from 0°* ; a southeast wind, a *wind from 135°* , etc.

Problems

1. Taking a convenient scale, indicate graphically the following wind velocities and directions: (a) 35 miles from 70° ; (b) 15 miles from 160° ; (c) 40 miles from 340° ; (d) 10 miles from 250° .

2. A plane has an air speed of 150 mi./hr.; there is a 40-mile wind from 315° . Find the ground speed, track, and drift for the following headings, by constructing a figure like that of Figure 25. (Do not write in your text.)

Heading	Ground Speed	Track	Drift
315°	-----	-----	-----
0°	-----	-----	-----
45°	-----	-----	-----
90°	-----	-----	-----
135°	-----	-----	-----

3. A plane has an air speed of 150 mi./hr.; there is a 40-mile wind from 315° . Find the heading, ground speed, and drift to fly the following tracks, by constructing figures like that of Figure 26.

Heading	Ground Speed	Track	Drift
-----	-----	315°	-----
-----	-----	340°	-----
-----	-----	40°	-----
-----	-----	80°	-----
-----	-----	115°	-----

4. A plane has an air speed of 200 mi./hr.; there is a 30-mile wind from 210° . Complete the following table by finding heading or track, ground speed, and drift.

Heading	Ground Speed	Track	Drift
-----	-----	0°	-----
0°	-----	-----	-----
-----	-----	30°	-----
30°	-----	-----	-----
60°	-----	-----	-----
-----	-----	60°	-----
90°	-----	-----	-----
-----	-----	90°	-----
120°	-----	-----	-----

5. What heading shall a plane take in order to track 225° , if there is a 30-mile wind from 170° ? The air speed is 130 mi./hr.

6. There is a wind of 25 miles per hour from a direction of 320° . Find the track, drift angle, and ground speed of a plane heading 60° with an air speed of 120 mi./hr.

7. Find the heading for a plane to track 310° , if the wind is 20 mi./hr. from 40° and the air speed 90 knots.

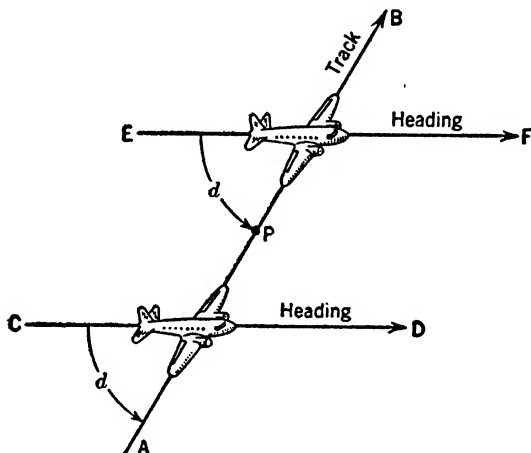


FIGURE 27.

The Wind Star. In the problems of the previous section, it was assumed that the wind direction and velocity were known to the navigator. In many cases it is necessary to determine the heading for a given track without a knowledge of the wind's velocity and direction. This problem may be solved by measuring the drift on two headings.

In order to measure the drift it is necessary to sight back on some fixed point on the surface, over which the plane has passed. Figure 27 shows two positions of a plane following a track AB, with a heading represented by the parallel lines CD and EF. The drift angle is the angle between the fore-and-aft axis of the plane and the track, angle d in Figure 27. Angle d may be measured by sighting back on some fixed point like P over which the plane has passed. In order to measure drift on ocean flights, flares have been developed which will burn for some time after they have struck the water. For daylight flying, Pan American Airways developed a special "bomb" filled with aluminum powder which forms a visible mark after it strikes the water.

The *wind star* is a graphic method of finding the heading for a required track from measurements of drift on two headings.

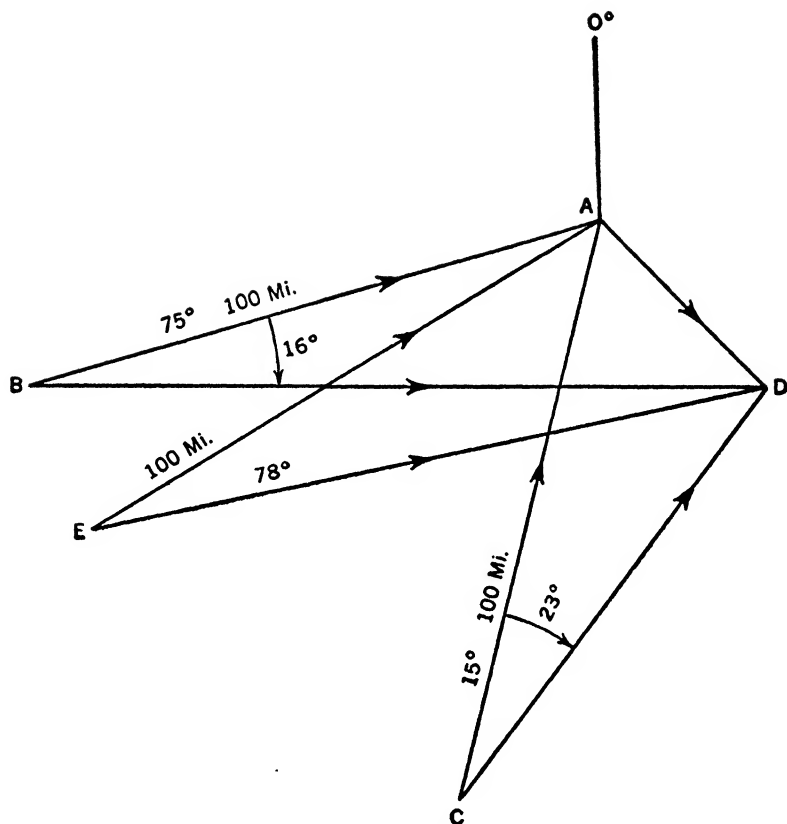


FIGURE 28. THE WIND STAR.

Example. A plane with an air speed of 100 mi./hr. is to be flown on a track of 78° . By trial it has been found that the drift is 23° to the right, on a heading of 15° , and 16° to the right on a heading of 75° .

This problem may be solved graphically by plotting the reciprocal bearings (see page 25). In Figure 28 draw AB with a bearing of 255° and a length of 100 miles. Since the bearing of AB is 255° , then the bearing of BA = $255^\circ - 180^\circ = 75^\circ$. Draw the indefinite line BD, making angle ABD = 16° . Next construct AC with a length of 100 miles, and a bearing of 195° ($15^\circ + 180^\circ$); lay off angle ACD = 23° . Extend BD and CD to intersect at point D.

If the plane started from B, in calm air, it would arrive at A in one hour; but since it has a 16° right drift on this heading, it follows the track BD. Similarly, if the plane started from C, with no wind, it would reach A in one hour; actually starting from C it will follow the track CD due to the

wind. Hence the wind direction and velocity are represented by the bearing and length of AD.

To determine the heading which the plane must take to follow the required track of 78° , construct DE with a bearing of 258° , so that ED represents the track of 78° . Then, with A as a center, and a radius of 100 miles, the air speed, draw an arc intersecting DE at E.

The required heading is the bearing of EA, 58° .

The drift angle is angle AED, 20° .

The ground speed is the length of ED, 113 miles.

This method may be summarized by noting that the bearings of all lines drawn to A represent headings, and that the bearings of all lines drawn to D represent tracks. The lengths of all lines drawn to A represent air speed; and the lengths of all lines drawn to D represent ground speeds. There are various means for solving the wind star mechanically.

Problems

1. An airplane making 120 mi./hr. air speed drifts 15° to the left on a heading of 140° , and 8° to the right on a heading of 40° . Find the heading, ground speed, and angle of drift for a track of 95° .

2. It is desired to fly due west with a plane having an air speed of 80 mi./hr. On a heading of 300° the drift is 9° to the right and on a heading of 240° , 21° to the right. Find the heading, ground speed, and drift angle.

3. Find the heading to track 38° , with a plane making 200 mi./hr. air speed, if the drift is 15° left on a heading of 88° , and 17° left on a heading of 68° .

4. A plane which has an air speed of 175 knots drifts 6° right on a heading of 265° , and 10° right on a heading of 306° . What heading should the plane take in order to track 302° ? What ground speed will it have on this heading?

5. It is desired to track 116° with a plane making 200 mi./hr. air speed. The drift is 12° right on a 90° heading, and 15° right on a 60° heading. What heading should the plane take and what ground speed will it have?

6. The air speed of a plane is 150 knots. It drifts 10° to the right on a heading of 290° , and 6° to the right on a heading of 260° . Find the heading, ground speed, and drift for the plane to track 253° .

7. A plane with an air speed of 200 knots heads 53° and drifts 6° to the left; on a heading of 81° it drifts 8° to the left. Find the heading, drift, and ground speed for a track of 102° .

Interception. There is an old rule in navigation which says that "constant bearing means collision." This means that if the bearing of one ship from another is constant, the ships will collide. In Figure 29, let $A, A', A'',$ etc. represent different positions of a ship following the course AC ; and $B, B', B'',$ etc. the corresponding positions of a second ship following the course BC . If the bearing of the second ship from the first remains constant, then the line connecting the two ships is always parallel to the base AB , and the ships must reach C at the same time.

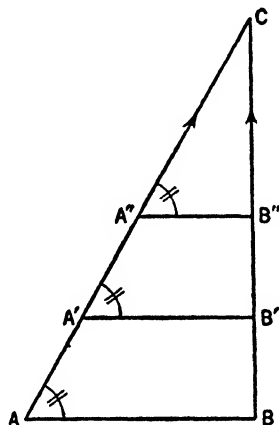


FIGURE 29.

In air navigation the rule about constant bearing may be changed to "constant bearing means interception." The problem of interception has many obvious uses; planes are required to meet for patrol, to intercept hostile aircraft or ships, or to return to a carrier after a flight. The problem of interception is solved graphically.

Example 1. Interception without wind and in less than one hour.

Point A (Figure 30) is 60 miles south and 70 miles east of point B. A plane leaves A on a track of 280° with a speed of 150 mi./hr. At the same time, a second plane with a speed of 130 mi./hr. leaves B. What is the track of the second plane; when and where will the interception occur?

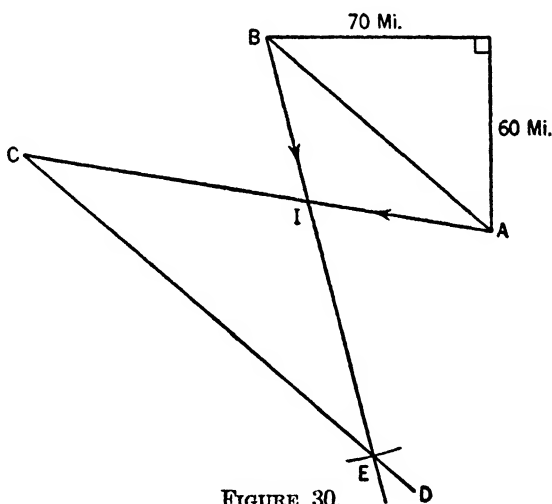


FIGURE 30.

Since there is no wind, track and heading are the same; and air speed and ground speed are the same. Construct the line AC with a bearing of 280° and a length of 150 miles. Through C draw CD parallel to BA. With B as a center and a radius of 130 miles, draw an arc intersecting CD at E. BE represents the track and speed of the plane from B. The interception occurs at point I. The problem may be checked by finding the number of hours required for each plane to reach point I.

Time for plane from B: $BI/130 = 52/130 = .40$ hour.

Time for plane from A: $AI/150 = 60/150 = .40$ hour.

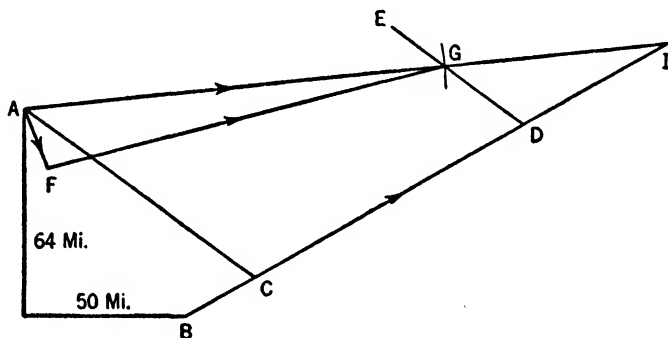


FIGURE 31.

Example 2. Interception with wind and after one hour.

Point B (Figure 31) is 50 miles east and 64 miles south of point A. A plane is observed passing over point B on a track of 60° with a ground speed of 100 mi./hr. There is a 20-mile wind from 340° . Fifteen minutes later a plane is dispatched from A, which has an air speed of 130 mi./hr. Find the heading, track, and ground speed required for the plane from A to intercept the plane from B.

Construct the line BI with a bearing of 60° to represent the track of the plane from B. Make $BC = 25$ miles, the distance covered by the first plane in the 15-minute interval; then lay off $CD = 100$ miles, the distance covered by the first plane in one hour. Draw DE parallel to CA. Construct AF to represent the wind in one hour, 20 miles from 340° . With F as a center and a radius equal to 130 miles, draw an arc intersecting DE at G. AFG is the usual wind triangle for the plane from A; FG representing heading and air speed; AG the track and ground speed. Extend AG to intersect BI at point I. I is the point of interception.

The heading of the pursuing plane is the bearing of FG, 75° .

The track of the pursuing plane is the bearing of AG, 84° .

Its ground speed is the length of AG, 134 mi./hr.

Time for plane from A to reach I = $AI/AG = 203/134 = 1.51$ hours.

Time for plane from B to reach I = $BI/100 = 176/100 = 1.76$ hours.

The difference of .25 hours between the two times is the fifteen-minute start of the plane from B.

The problem of interception depends on the intersection of a line and a circle. In example 2, the circle, with center F and radius 130, intersects the line DE. In some cases the circle will not intersect the line, and then no interception is possible. In other cases there may be two intersections; the two intersections, however, do not always give two solutions for the problem of interception.

Problems

1. In example 1, change the heading of the plane from A to 252° . Show that interception takes place in one hour.

2. Show that interception is impossible in example 1 if the heading of the plane from A is changed to 240° .

3. Point A is 59 miles east and 70 miles north of B. A plane leaves A on a heading of 250° at 140 mi./hr. air speed. At the same time a plane leaves B at 110 mi./hr. There is no wind. Find the heading of the plane from B and the time of interception.

4. B is 60 miles east and 90 miles south of A. There is a 40-mile wind from 315° . A plane from B heads 60° at 120 knots air speed. Fifteen minutes later a plane leaves A at 175 knots air speed. Find the tracks and ground speeds of the two planes and the time required for interception.

5. A plane is sighted over Montauk Point on Long Island, heading 240° and making 150 miles per hour. Fifteen minutes later a plane is to be dispatched from Mitchel Field to intercept the first plane. The pursuing plane is also capable of making 150 miles per hour. Mitchel Field is 80.9 miles east and 19 miles south of Montauk Point. Find the heading of the pursuing plane and how far from Mitchel Field the interception takes place.

6. A is 95 miles north and 45 miles east of B. A plane is to start from B on a track of 300° and has a ground speed of 90 miles an hour. At the same time a plane is to take off from A to join the plane from B. The plane from A has an air speed of 130 miles an hour. There is a 26-mile wind from the direction of 40° . What heading should the plane from A take, and where will the interception occur?

7. B is 74 miles north and 51 miles west of A. A plane leaves A and flies due east at 90 miles an hour. At the same time a second plane making 120 miles per hour leaves B to overtake the first plane. There is no wind. Find the heading the pursuing plane should take, and determine where the interception occurs.

8. Solve problem 7 if there is a 30-mile wind from 225° .

Radius of Action Returning to a Fixed Base. It is frequently necessary to determine how far a plane can fly from a given point, and return within a fixed time. The total time which is allowed for flight depends upon the fuel supply. If there is no wind, the distance the plane can fly is determined at once by the air speed, and the "time out" will be equal to the "time in." When there is a wind, the ground speed on the outbound trip will be different from the ground speed of the return flight. In this case it is necessary to determine these two ground speeds, and use them to calculate the distance, called *radius of action*, which the plane may safely fly.

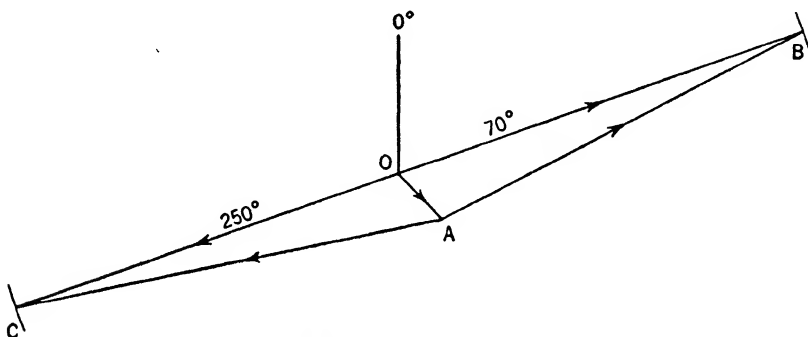


FIGURE 32. RADIUS OF ACTION — FIXED BASE.

Example. A plane is ordered to leave point O (Figure 32) and scout in a direction 70° . The plane has an air speed of 225 mi./hr., and the total time of flight is to be three hours. There is a 30-mile wind from 315° . Find the radius of action.

The first step is to find the ground speeds for the track out (70°) and the track in (250°), under the given wind conditions. These two triangles may be constructed together as is shown in Figure 32. Draw OA to represent the wind, and the line COB to represent the tracks of 70° and 250° . From A as a center, and with a radius equal to the air speed of 225 mi./hr., draw arcs intersecting CB at C and B.

The ground speed out or $g_1 = 240$ mi./hr.

The ground speed in or $g_2 = 214$ mi./hr.

Denote the time out by t_1 , the time in by t_2 , the total time by T , and the radius of action by R , then:

$$t_1 = \frac{R}{g_1}, \quad t_2 = \frac{R}{g_2}$$

$$T = t_1 + t_2 = \frac{R}{g_1} + \frac{R}{g_2}$$

$$T = \frac{R(g_1 + g_2)}{g_1 g_2}$$

$$R = \frac{T g_1 g_2}{g_1 + g_2}$$

For this problem, $T = 3$ hours, $g_1 = 240$ mi./hr., $g_2 = 213$ mi./hr., and

$$R = \frac{3(240)(213)}{453} = 339 \text{ miles}$$

$$t_1 = \frac{T g_2}{g_1 + g_2} = \frac{3(213)}{453} = 1.41 \text{ hours}$$

$$t_2 = \frac{T g_1}{g_1 + g_2} = \frac{3(240)}{453} = 1.59 \text{ hours}$$

Hence the plane follows the track 70° for 1.41 hours with a ground speed of 240 mi./hr.; it turns and follows the return track of 250° for 1.59 hours with a ground speed of 213 mi./hr.

In all of the problems which involve wind velocity, the answer obtained is valid for the given wind conditions, and any change in the wind requires that the problem be reworked with the new data.

Problems

Find the radius of action for each of the following:

	<i>Total Time</i>	<i>Track Out</i>	<i>Air Speed</i>	<i>Wind</i>
1.	4 hours	30°	160 knots	40 miles from 90°
2.	4 hours	30°	160 knots	40 miles from 270°
3.	4 hours	210°	160 knots	40 miles from 45°
4.	3 hours	210°	160 knots	40 miles from 45°
5.	3 hours	330°	200 mi./hr.	20 miles from 220°
6.	2.5 hours	195°	175 mi./hr.	15 miles from 70°
7.	3 hours	130°	140 mi./hr.	20 miles from 100°
8.	2 hours	75°	200 knots	15 miles from 0°
9.	2 hours	255°	200 knots	15 miles from 180°

Radius of Action Returning to a Moving Base. When a plane leaves a carrier, the method of the last section does not apply, since the carrier will not remain in a fixed position. The problem of returning to a moving base is a combination of the simple wind triangle and the problem of interception.

Example. A carrier is following a course of 0° with a speed of 20 knots. There is a 25-knot wind from 310° . A plane with 90-knot air speed and fuel for 4 hours is ordered to scout on a track of 60° . How long may the plane scout in this direction, before changing course to return to the carrier?

Problems

1. A carrier is following a course of 130° at 25 knots. A plane with an air speed of 160 knots is ordered to patrol on a track of 30° , and return in four hours. There is a 30-knot wind from 90° . Find the time out, and the ground speeds out and in.

2. Answer problem 1 if the track out is 60° .

3. Answer problem 1 if the track out is 90° .

4. Answer problem 1 if the track out is 30° and the wind is 20 knots from 120° .

5. A carrier is following a course of 75° at 22 knots. A plane is to scout 3 hours on a track of 350° . The plane has an air speed of 120 knots and the wind is 15 knots from 270° . Find the time out, and the ground speeds out and in.

6. Answer problem 5 if the track is 10° .

7. Answer problem 5 if the wind is 15 knots from 225° .

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CHAPTER V

SPHERICAL TRIGONOMETRY OF THE EARTH

In Chapter I, the lengths of great circle arcs were found by using a "protractor" on a globe. This method was approximate only; even on a very large globe, it would be impossible to obtain an answer correct to the nearest minute. Finding an accurate answer for a great circle arc is a problem of spherical trigonometry. In problems of piloting, such as doubling the angle on the bow or finding the distance at which an object is passed abeam (page 28), the navigator is using a plane triangle to determine a line which he cannot measure. Spherical triangles on the earth, that is, triangles formed by great circles, are used in a similar manner to find great circle arcs.

Four methods of finding position have been mentioned; they are piloting, dead reckoning, radio navigation, and celestial navigation. Celestial navigation, which is finding latitude and longitude from observations of the sun and stars, also depends on the solution of spherical triangles.

In this chapter two fundamental formulas of spherical trigonometry will be developed and applied to earth problems. These two formulas are all that are necessary for finding great circle arcs, and for working the main problems of celestial navigation. There are many other formulas of spherical trigonometry which are not considered here. The computations of spherical trigonometry are made much shorter if a function called the *haversine* is used. The haversine function will be studied briefly before beginning the study of spherical trigonometry.

The Haversine Function. The sine, cosine, and other functions of an angle may be defined both as ratios and lines. For either of the angles marked A in Figure 34

$$\sin A = y/r \quad \text{and} \quad \cos A = x/r.$$

If $r = 1$, then

$$\sin A = y \quad \text{and} \quad \cos A = x.$$

The distance BD in Figure 34 is $r - x$, or if $r = 1$,

$$BD = 1 - x = 1 - \cos A.$$

BD, like the sine and cosine, is a function of angle A. It was formerly

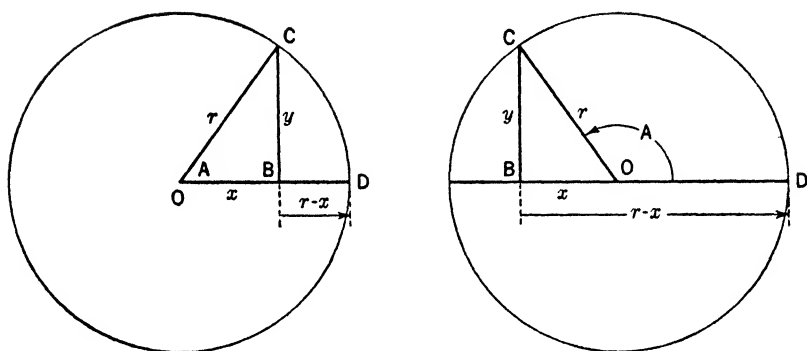


FIGURE 34.

called the *sagitta*, or arrow, since if CB is extended to intersect the circle, the figure resembles a bow and arrow. Later BD was called the *versed sine*.

It is convenient to take one half of BD. BD itself will vary from 0 to 2; and hence BD/2 varies from 0 to 1. The function BD/2 is called the *haversine*, half of the *versed sine*, and is abbreviated *hav*. Hence

$$\text{haversine } A = \text{hav } A = \frac{1 - \cos A}{2}$$

Both logarithmic and natural haversines are used in solving spherical triangles. Haversines may be found from the known values of the natural cosines.

$$\text{hav } 0^\circ = \frac{1 - \cos 0^\circ}{2} = \frac{1 - 1}{2} = 0$$

$$\text{hav } 60^\circ = \frac{1 - \cos 60^\circ}{2} = \frac{1 - .50}{2} = .25$$

$$\text{hav } 90^\circ = \frac{1 - \cos 90^\circ}{2} = \frac{1 - 0}{2} = .50$$

$$\text{hav } 120^\circ = \frac{1 - \cos 120^\circ}{2} = \frac{1 + .50}{2} = .75$$

$$\text{hav } 180^\circ = \frac{1 - \cos 180^\circ}{2} = \frac{1 + 1}{2} = 1$$

$$\log \text{hav } 60^\circ = \log .25 = 9.39794$$

$$\log \text{hav } 90^\circ = \log .50 = 9.69897$$

$$\log \text{hav } 120^\circ = \log .75 = 9.87506$$

The haversine increases from 0 to 1 as the angle increases from 0° to 180° ; the haversines of all angles are positive and never greater than 1.

The haversine is a means of simplifying certain formulas of plane trigonometry. For example, for $\sin \frac{x}{2} = \sqrt{\frac{1 - \cos x}{2}}$, one may write

$\sin \frac{x}{2} = \sqrt{\text{hav } x}$. The formula $\sin \frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$ is used to find angle A in triangle ABC when the three sides are given. The square root may be avoided by writing, $\text{hav } A = \frac{(s-b)(s-c)}{bc}$.

Problems

Solve the following problems without using the haversine tables. Find the natural haversines from the given natural cosines. (Do not write in your text.)

1. Prove that $\text{hav}(-x) = \text{hav } x$ and that $\text{hav}(360^\circ - x) = \text{hav } x$.
2. Complete the following table:

x	$\text{Cos } x$	$\text{Hav } x$
0°	1.000	-----
30°	.866	-----
45°	.707	-----
60°	.500	-----
90°	.000	-----
120°	-.500	-----
135°	-.707	-----
150°	-.866	-----
180°	-1.000	-----

3. From the results of problem 2 find the values of the haversines of 210° , 225° , 240° , 270° , 300° , 315° , 330° without further computation.

4. Use the results of problems 2 and 3 to plot the graph of $y = \text{hav } x$ from $x = 0$ to $x = 360^\circ$.

5. Complete the following:

x	$\text{Cos } x$	$\text{Hav } x$	$\text{Log Hav } x$
$3^\circ 37'$.99801	-----	-----
$3^\circ 38'$.99799	-----	-----
$11^\circ 28'$.98004	-----	-----
$11^\circ 29'$.97998	-----	-----
$36^\circ 52'$.80003	-----	-----
$36^\circ 53'$.79986	-----	-----

6. From the results of problem 5 complete the following.

(a) Log haversines of angle between $36^\circ 53'$ and 180° have a characteristic of -----.

(b) Log haversines of angles between ----- and ----- have a characteristic of $8 - 10$.

(c) Log haversines of angles between and have a characteristic of 7 - 10.

7. Complete the following table.

x	$\cos x$	$\text{Log Hav } x$	$\text{Hav } x$
$53^\circ 00'$.60182	-----	-----
$53^\circ 10'$.59949	-----	-----
$53^\circ 20'$.59716	-----	-----

The Haversine Table. Pages xliv-xlviii contain a table of haversines for angles from 0° to 180° , at ten-minute intervals. The angle, its log haversine, and the natural haversine, are printed in adjacent columns. On page xlv the first angle is $40^\circ 00'$, printed 40° . Hence from page xlv:

$$\begin{aligned}\log \text{hav } 40^\circ 00' &= 9.06810 \\ \text{nat hav } 40^\circ 00' &= .11698 \\ \log \text{hav } 40^\circ 10' &= 9.07157 \\ \text{nat hav } 40^\circ 10' &= .11791\end{aligned}$$

On page xlv the characteristic of the log haversine varies from 4 - 10 to 9 - 10; the characteristic is printed for each angle on this page. On pages xlv-xlviii all the characteristics (except for the three last entries) are 9 - 10 and are printed only in the headings of the columns. In using the haversine table, it is necessary to interpolate to the nearest minute.

Example 1. Find $\log \text{hav } 36^\circ 54'$ and $\text{hav } 36^\circ 54'$.

To find the log haversine $36^\circ 54'$, find the difference of $\log \text{hav } 36^\circ 50'$ and $\log \text{hav } 37^\circ 00'$, multiply this difference by .4, and add to $\log \text{hav } 36^\circ 50'$. To find $\text{nat hav } 36^\circ 54'$, subtract the natural haversines of $36^\circ 50'$ and $37^\circ 00'$, multiply the difference by .4, and add to the natural haversine $36^\circ 50'$. This process of interpolation may be represented as follows.

Angle	Log Hav	Nat Hav
$37^\circ 00'$	9.00295	.10068
$\left[\begin{array}{l} 36^\circ 54' \\ 4' \quad 10' \\ 36^\circ 50' \end{array} \right]$	$\left[\begin{array}{l} 9.00068 \\ x \quad 378 \\ 8.99917 \end{array} \right]$	$\left[\begin{array}{l} .10016 \\ y \quad 87 \\ .09981 \end{array} \right]$
	$\frac{4}{10} = \frac{x}{378}$	$\frac{4}{10} = \frac{y}{87}$
	$x = 151$	$y = 35$

This diagram to represent interpolation is intended only as a means of making the process clear. Frequently the arithmetic involved in interpolation can be performed mentally; in any case the written work of interpolation should be reduced to a minimum.

Example 2. Find angle x if $\log \text{hav } x = 9.04270$.

From page xliv 9.04270 is between the log haversines of $38^\circ 50'$ and $38^\circ 40'$.

$$\log \text{hav } 38^\circ 50' - \log \text{hav } 38^\circ 40' = .00359$$

$$\log \text{hav } x - \log \text{hav } 38^\circ 40' = .00288$$

$$\frac{.00359}{.00288} \times 10' = 8'$$

$$x = 38^\circ 40' + 8' = 38^\circ 48'$$

To clarify the process, the example may be represented in a manner similar to example 1.

Angle		Log Hav			
$38^\circ 50'$] 10'	9.04341] 359		
x		9.04270			
y		288			
$38^\circ 40'$		9.03982			
$\frac{y}{10} = \frac{288}{359}$					
$y = 8'$					

Problems

Use the haversine table to solve the following problems.

1. Find the log and natural haversines of the following angles:

- (a) $110^\circ 32'$, (b) $56^\circ 47'$, (c) $11^\circ 25'$, (d) $15^\circ 18'$, (e) $97^\circ 45'$, (f) $3^\circ 34'$,
(g) $27^\circ 33'$, (h) $36^\circ 57'$, (i) $119^\circ 43'$, (j) $179^\circ 57'$.

2. Find the angle corresponding to the following log haversines:

- (a) 9.18998, (b) 9.90621, (c) 8.23331, (d) 9.91067, (e) 6.93166, (f) 9.99806,
(g) 7.14506, (h) 8.10216, (i) 9.11227, (j) 9.45725.

3. Find the angle corresponding to the following natural haversines:

- (a) .80594, (b) .97994, (c) .00186, (d) .76495, (e) .63472, (f) .01579,
(g) .64693, (h) .84231, (i) .00942, (j) .79832.

The Haversine Formula for Great Circle Arcs. In plane trigonometry the Law of Cosines, $c^2 = a^2 + b^2 - 2ab \cos C$, is used to find the third side of a triangle when the other two sides and their included angle are given. The purpose of this section is to develop a formula for finding the third side of a spherical triangle when two sides and the included angle are given.

Spherical triangles are formed on the surface of a sphere by great circle arcs. The sides and angles of spherical triangles are both measured in degrees and minutes (see Triangles on the Earth, pages 8-9).

Plane triangles are used in proving relationships between the sides and angles of spherical triangles.

Figure 35 represents a spherical triangle drawn on a sphere with center O and radius r . The sides BC , CA , and AB are denoted by a , b , and c . The plane triangle CDE is formed by drawing the tangents to AC and BC at C ; these tangents meet OA and OB extended at D and E . The four plane triangles in Figure 35 are drawn separately in Figure 36. Angle DOC equals the arc AC or b ; angle OCD is a right angle; hence in triangle COD ,

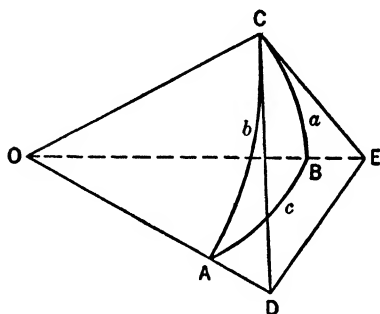


FIGURE 35.

$$CD = r \tan b \quad \text{and} \quad OD = r \sec b$$

Similarly in triangle COE ,

$$CE = r \tan a \quad \text{and} \quad OE = r \sec a$$

Angle C , in triangle CDE , is the angle between the two tangents and is equal to angle C in the spherical triangle. Angle EOD equals the arc AB or c .

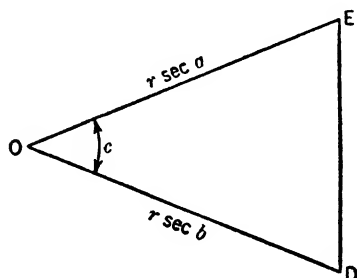
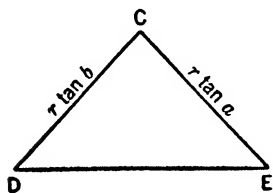
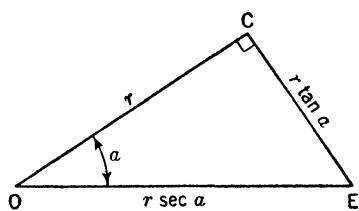
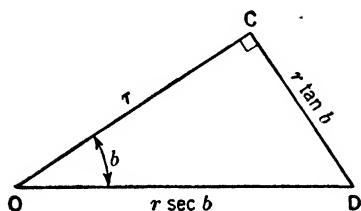


FIGURE 36.

Two sides of both triangles CDE and EOD are known from the right triangles OCD and COE. Applying the Law of Cosines to EOD,

$$\overline{DE}^2 = r^2 \sec^2 a + r^2 \sec^2 b - 2 r^2 \sec a \sec b \cos c. \quad (1)$$

Applying the Law of Cosines to CDE,

$$\overline{DE}^2 = r^2 \tan^2 a + r^2 \tan^2 b - 2 r^2 \tan a \tan b \cos C. \quad (2)$$

Subtracting (2) from (1) and dividing by r^2 ,

$$0 = \sec^2 a - \tan^2 a + \sec^2 b - \tan^2 b - 2 \sec a \sec b \cos c + 2 \tan a \tan b \cos C.$$

Applying the formulas $\sec^2 a - \tan^2 a = 1$ and $\sec^2 b - \tan^2 b = 1$,

$$0 = 2 - 2 \sec a \sec b \cos c + 2 \tan a \tan b \cos C.$$

Transposing and dividing by 2,

$$\sec a \sec b \cos c = 1 + \tan a \tan b \cos C.$$

Multiplying both sides by $\cos a \cos b$,

$$\cos c = \cos a \cos b + \sin a \sin b \cos C. \quad (3)$$

Formula (3) is called the *sine-cosine formula* and may be used to find side c from a , b , and C ; or to find angle C , if the three sides are given. In the sine-cosine formula, $\cos c$ is the sum of two products. Either product may be positive or negative; $\cos a \cos b$ is negative when either a or b is obtuse; $\sin a \sin b \cos C$ is negative when C is obtuse.

The sine-cosine formula is not well adapted for computation by logarithms. Since $\text{hav } c = \frac{1 - \cos c}{2}$, $\cos c$ may be replaced by $1 - 2 \text{hav } c$, and $\cos C$ by $1 - 2 \text{hav } C$. A shorter and more convenient form is easily obtained by making these substitutions for $\cos c$ and $\cos C$.

$$1 - 2 \text{hav } c = \cos a \cos b + \sin a \sin b (1 - 2 \text{hav } C).$$

$$1 - 2 \text{hav } c = \cos a \cos b + \sin a \sin b - 2 \sin a \sin b \text{hav } C.$$

$$\text{But } \cos a \cos b + \sin a \sin b = \cos (a - b) = 1 - 2 \text{hav } (a - b).$$

$$\text{Hence, } 1 - 2 \text{hav } c = -2 \sin a \sin b \text{hav } C + 1 - 2 \text{hav } (a - b).$$

This reduces to

$$\text{hav } c = \sin a \sin b \text{hav } C + \text{hav } (a - b)$$

the *haversine formula* for spherical triangles.

Problems

1. Prove that $\cos b = \cos a \cos c + \sin a \sin c \cos B$ and that $\cos a = \cos b \cos c + \sin b \sin c \cos A$.

2. Prove that $\text{hav } b = \sin a \sin c \text{hav } B + \text{hav } (a - c)$ and that $\text{hav } a = \sin b \sin c \text{hav } A + \text{hav } (b - c)$.

3. Find formulas for $\cos C$, $\cos B$, $\cos A$ from the sine-cosine formula.

4. Find formulas for $\text{hav } C$, $\text{hav } B$, and $\text{hav } A$ from the haversine formula.

Great Circle Distances on the Earth. When the latitudes and longitudes of two places are given, the haversine formula may be used to find the great circle arc which joins them. If the two latitudes are denoted by L_1 and L_2 , the difference of longitude by DLo , and the great circle arc by D ; then the haversine formula becomes

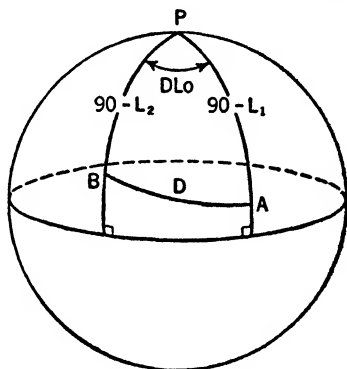


FIGURE 37

This formula is easily obtained from the haversine formula of the last section.

In Figure 37 triangle PAB is formed by two points and the pole.

$$PB = 90^\circ - \text{latitude of B} = 90^\circ - L_2$$

$$PA = 90^\circ - \text{latitude of A} = 90^\circ - L_1$$

$$P = \text{difference of longitude} = DLo$$

$$AB = \text{required arc} = D$$

$$\sin (PA) = \sin (90^\circ - L_1) = \cos L_1$$

$$\sin (PB) = \sin (90^\circ - L_2) = \cos L_2$$

$$\text{hav } (PB - PA) = \text{hav } (90^\circ - L_2 - 90^\circ + L_1) = \text{hav } (L_1 - L_2)$$

Applying the formula $\text{hav } c = \sin a \sin b \text{hav } C + \text{hav } (a - b)$ to triangle PAB,

$$\text{hav } D = \cos L_1 \cos L_2 \text{hav } DLo + \text{hav } (L_1 - L_2)$$

Example 1. Find the great circle distance between San Francisco ($37^\circ 50' \text{ N.}, 122^\circ 26' \text{ W.}$) and Tokyo ($35^\circ 39' \text{ N.}, 139^\circ 45' \text{ E.}$).

$$L_1 = 37^\circ 50', L_2 = 35^\circ 39', L_1 - L_2 = 2^\circ 11'$$

$$DLo = 360^\circ - (122^\circ 26' + 139^\circ 45') = 97^\circ 49'$$

$$\text{hav } D = \cos 37^\circ 50' \cos 35^\circ 39' \text{hav } 97^\circ 49' + \text{hav } 2^\circ 11'$$

log cos L_1	$(37^\circ 50')$	= 9.89752
+ log cos L_2	$(35^\circ 39')$	= 9.90987
log hav DLo	$(97^\circ 49')$	= 9.75435
		9.56174

antilog 9.56174	= .36453
-----------------	----------

+ nat hav $(L_1 - L_2) (2^\circ 11')$	= .00036
---------------------------------------	----------

nat. hav D	= .36489
--------------	----------

$$D = 74^\circ 20'$$

$$D = 4460 \text{ nautical miles}$$

In using the haversine formula, *south latitudes are negative*. DLo is either the sum, difference, or 360° minus the sum of the longitudes (see DLo under Parallel Sailing, pages 19–20). All the functions in the haversine formula are positive, even when one or both of the latitudes are negative.

Example 2. Find the great circle distance between San Francisco and Tahiti ($17^\circ 30' \text{ S.}$, $149^\circ 30' \text{ W.}$).

$$\begin{aligned}
 L_1 &= 37^\circ 50', & L_2 &= -17^\circ 30' \\
 L_1 - L_2 &= 37^\circ 50' - (-17^\circ 30') = 55^\circ 17' \\
 \text{DLo} &= 149^\circ 30' - 122^\circ 26' = 27^\circ 04' \\
 \cos L_2 &= \cos (-17^\circ 30') = \cos 17^\circ 30' \\
 \log \cos L_1 & & (37^\circ 50') &= 9.89781 \\
 + \log \cos L_2 & & (17^\circ 30') &= 9.97942 \\
 \log \text{hav DLo} & & (27^\circ 04') &= 8.73847 \\
 & & & \underline{8.61570} \\
 \text{antilog } 8.61570 & & &= .04128 \\
 \text{nat hav } (L_1 - L_2) (55^\circ 17') & & &= .21524 \\
 & \text{nat hav D} & &= .25652 \\
 & D & &= 60^\circ 52' \\
 & D & &= 3652 \text{ nautical miles}
 \end{aligned}$$

In some cases $L_1 - L_2$ is negative, but since $\text{hav}(-x) = \text{hav } x$, $\text{hav}(L_1 - L_2)$ is positive.

Problems

1. Show that the computation is unchanged in examples 1 and 2 if L_1 and L_2 are interchanged.

Find the great circle distance between the two given places in each of the following problems.

2. Harbour Grace ($47^\circ 43' \text{ N.}$, $53^\circ 08' \text{ W.}$) to Moscow ($55^\circ 45' \text{ N.}$, $37^\circ 34' \text{ E.}$).

3. New York ($40^\circ 43' \text{ N.}$, $74^\circ 00' \text{ W.}$) to Cape of Good Hope ($34^\circ 21' \text{ S.}$, $18^\circ 30' \text{ E.}$).

4. From Manila ($14^\circ 36' \text{ N.}$, $120^\circ 57' \text{ E.}$) to Hong Kong ($22^\circ 16' \text{ N.}$, $114^\circ 10' \text{ E.}$).

5. From Oporto Light ($41^\circ 09' \text{ N.}$, $8^\circ 41' \text{ W.}$) to Block Island Light ($41^\circ 09' \text{ N.}$, $71^\circ 33' \text{ W.}$). Compare this answer with the distance along the parallel of $41^\circ 09' \text{ N.}$

6. From Port Elizabeth ($33^\circ 58' \text{ S.}$, $25^\circ 37' \text{ E.}$) to Freemantle ($32^\circ 03' \text{ S.}$, $115^\circ 44' \text{ E.}$).

7. Auckland ($36^\circ 50' \text{ S.}$, $174^\circ 46' \text{ E.}$) to Sitka ($57^\circ 03' \text{ N.}$, $135^\circ 20' \text{ W.}$).

8. Pitcairn Island ($25^{\circ} 04' \text{ S.}$, $130^{\circ} 08' \text{ W.}$) to Easter Island ($27^{\circ} 10' \text{ S.}$, $109^{\circ} 26' \text{ W.}$).

9. Lakehurst ($40^{\circ} 01' \text{ N.}$, $74^{\circ} 20' \text{ W.}$) to Frankfurt-am-Main ($50^{\circ} 07' \text{ N.}$, $8^{\circ} 39' \text{ E.}$).

10. White Island, New Zealand ($37^{\circ} 30' \text{ S.}$, $177^{\circ} 11' \text{ E.}$) to Cape Howe, Victoria ($37^{\circ} 30' \text{ S.}$, $149^{\circ} 59' \text{ E.}$). Compare with example 1, page 20.

11. Choshi, Japan ($35^{\circ} 42' \text{ N.}$, $140^{\circ} 52' \text{ E.}$) to Cape Horn ($55^{\circ} 59' \text{ S.}$, $67^{\circ} 16' \text{ W.}$).

12. If the great circle arc of problem 11 is continued, it will intersect the coast of Africa at ($8^{\circ} 10' \text{ N.}$, $13^{\circ} 06' \text{ W.}$) between Cape St. Ann and Cape Sierra Leone, making one of the longest possible great circle courses entirely over water. Find the distance from Choshi to the point on the African coast.

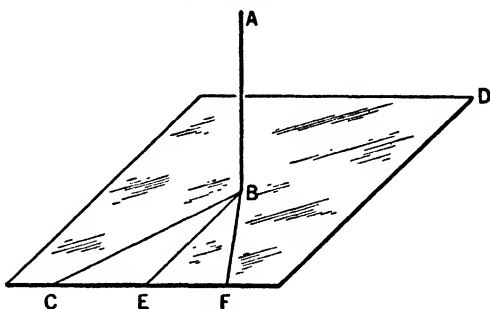


FIGURE 38.

The Sine Formula for Spherical Triangles. Angles in a spherical triangle may be found by a formula which is similar to the Law of Sines for plane triangles. Plane triangles are used in proving this sine formula in much the same way as they were used in proving the haversine formula.

The proof of the sine formula depends upon a theorem of solid geometry which is illustrated in Figure 38. If a given line is perpendicular to a plane, it is perpendicular to every line in the plane which passes through the foot of the given line. In Figure 38, AB is perpendicular to the plane CD; the theorem states that AB is also perpendicular to BC, BE, BF, etc.

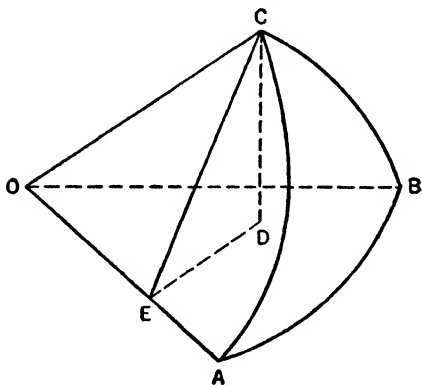


FIGURE 39.

In Figure 39, ABC is any spherical triangle drawn on a sphere with center O. Two plane right triangles are formed by the following constructions. Draw CD perpendicular to the plane OAB; construct the plane CDE perpendicular to OA. From the

theorem stated above, CDE and OCE are right triangles. Angle A in the spherical triangle is the angle between the tangents to CA and BA at A; the lines CE and DE are parallel to these tangents. Hence CED equals angle A.

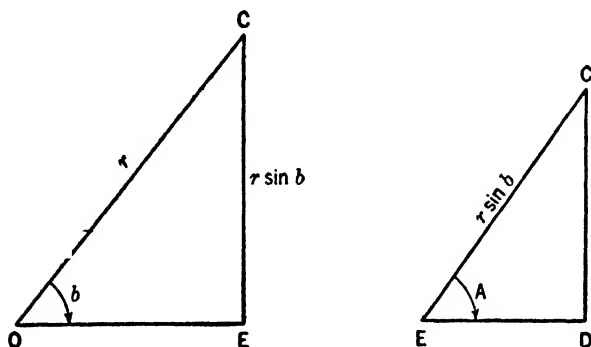


FIGURE 40.

In Figure 40 the right triangles COE and CED are drawn separately.

From COE, $CE = r \sin b$.

From CED, $CD = \overline{CE} \sin A = r \sin b \sin A$.

If a plane is constructed through CD perpendicular to OB, then it could be proved that

$$CD = r \sin a \sin B.$$

Hence, $CD = r \sin b \sin A = r \sin a \sin B$.

Then, $\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b}$

Since A and B are any two angles of ABC,

$$\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}$$

This is the *sine formula* for spherical triangles. The sine formula may be used to find the angles A and B in Figure 37, after the arc D has been found.

$$\frac{\sin A}{\cos L_2} = \frac{\sin B}{\cos L_1} = \frac{\sin DLo}{\sin D}$$

$$\sin A = \frac{\sin DLo \cos L_2}{\sin D} \quad \sin B = \frac{\sin DLo \cos L_1}{\sin D}$$

Example 1. Find the angles in the spherical triangle formed by the pole, San Francisco, and Tokyo.

Let A be the angle at San Francisco, and B the angle at Tokyo. From example 1, page 53, $L_1 = 37^\circ 50'$, $L_2 = 35^\circ 39'$, $DLo = 97^\circ 49'$, $D = 74^\circ 20'$.

$$\begin{aligned}\sin A &= \frac{\sin DLo \cos L_2}{\sin D} & \sin B &= \frac{\sin DLo \cos L_1}{\sin D} \\ &= \frac{\sin 97^\circ 49' \cos 35^\circ 39'}{\sin 74^\circ 20'} & &= \frac{\sin 97^\circ 49' \cos 37^\circ 50'}{\sin 74^\circ 20'} \\ \log \sin DLo \quad (97^\circ 49') &= 9.99595 \\ - \log \sin D \quad (74^\circ 20') &= 9.98356 \\ \log \text{quotient} &= 0.01239 \\ + \log \cos L_2 \quad (35^\circ 39') &= 9.90987 \\ \log \sin A &= 9.92226 \\ A &= 56^\circ 44' \\ \log \text{quotient} &= 0.01237 \\ \log \cos L_1 \quad (37^\circ 50') &= 9.89752 \\ \log \sin B &= 9.90989 \\ B &= 54^\circ 21'\end{aligned}$$

Since $\sin A = \sin (180^\circ - A)$, the sine function cannot distinguish between acute angles and their supplements. In the above example $\log \sin A = 9.92226$, hence A equals either $56^\circ 44'$ or $180^\circ - 56^\circ 44'$. This ambiguity may be avoided by using other rules of spherical trigonometry. The haversine formula may be written

$$\text{hav } C = \frac{\text{hav } c - \text{hav } (a - b)}{\sin a \sin b}$$

and used to find the angles. However, it is seldom necessary to use other rules. If the triangle is marked out on a globe with the "great circle protractor" used in Chapter I, it is easy to tell whether the angles are acute or obtuse. In example 1, it is obvious from the globe that both of the angles are acute.

The angles found by the sine formula give the *bearing* of the great circle at A and B. However, the direction or bearing of the great circle changes continually. Only the *initial course* can be obtained from the values of A and B.

Example 2. Find the angles in the triangle formed by San Francisco (A), Tahiti (B), and the pole.

From example 2, page 54, $L_1 = 37^\circ 50'$, $L_2 = -17^\circ 30'$, $DLo = 27^\circ 04'$, $D = 60^\circ 52'$.

Here it is obvious that angle A is obtuse and B acute.

$$\sin A = \frac{\sin 27^\circ 04' \cos 17^\circ 30'}{\sin 60^\circ 52'} \qquad \sin B = \frac{\sin 27^\circ 04' \cos 37^\circ 50'}{\sin 60^\circ 52'}$$

$$\begin{array}{rcl} \log \sin DLo \quad (27^\circ 04') & = & 9.65804 \\ - \log \sin D \quad (60^\circ 52') & = & 9.94126 \\ \log \text{quotient} & = & 9.71678 \\ + \log \cos L_2 \quad (17^\circ 30') & = & 9.97942 \\ \log \sin A & = & 9.69620 \\ & A = 180^\circ - 29^\circ 47' = 150^\circ 13' \\ \log \text{quotient} & = & 9.71678 \\ + \log \cos L_1 \quad (37^\circ 50') & = & 9.89752 \\ \log \sin B & = & 9.61430 \\ & B = 24^\circ 18' \end{array}$$

Problems

Use the results of the problems on page 54 and find the angles of the spherical triangles formed by the places named and the north pole. In each case verify that the sum of the three angles is between 180° and 540° .

1. The triangle of problem 2.
2. The triangle of problem 3.
3. The triangle of problem 4.
4. The triangle of problem 5.
5. The triangle of problem 6.
6. The triangle of problem 7.
7. The triangle of problem 8.
8. The triangle of problem 9.

Bibliography

Great Circle Sailing: Dutton, Chapter VI; Bowditch, 105-108; Bradley, A. D., *Proceedings of the U. S. Naval Institute*, Feb. 1941, 175-178.

Great Circle Charts: Great circle charts (the gnomonic projection) show great circles as straight lines. Great circle charts of the North Atlantic, South Atlantic, North Pacific, South Pacific, and Indian Oceans are published by the U. S. Hydrographic Office. The U. S. Coast and Geodetic Survey publishes a *Great Circle Chart of the United States*.

Construction and Use of Great Circle Charts: Deetz and Adams, 37-38, 45-46, 140-145; Bradley, A. D., *The American Mathematical Monthly*, Dec. 1940, 694-699.

CHAPTER VI

NAUTICAL ASTRONOMY

For centuries, people living in the northern latitudes have known that the North Star appears to have a fixed position in the heavens. From very early times, observations of the North Star have been used to determine latitude and to fix directions on the earth. It is common knowledge that the stars appear to move across the sky from east to west, and that each star appears to describe a circle around the North Star. Certain stars are seen to rise and set each day, and others are always observed to be above the horizon. In reality this daily motion of the stars is caused by the turning of the earth on its axis. To an observer, however, it seems that the earth is fixed and that the rising and setting of the stars is caused by a rotation of the sky.

It was long believed that the earth was stationary and that the stars were fixed points on an immense sphere, having the same center as the earth. While it is now known that this sphere is purely imaginary, it is convenient to use it for many problems of astronomy and celestial navigation. The daily motion of the stars may be explained by a daily rotation of this imaginary sphere; the stars and planets may be located on its surface by co-ordinates, just as points on the earth are located by latitude and longitude.

The Celestial Sphere. The immense sphere on which the stars appear to be located is called the *celestial sphere*. If the polar axis of the earth, pp' in Figure 41 (page 60), is extended, it intersects the celestial sphere in two points P and P'. P and P' are the *north* and *south celestial poles*. The North Star, or Polaris, is very near the north celestial pole. The *celestial equator* is a great circle of the celestial sphere midway between the celestial poles. The celestial equator is marked EE' in Figure 41.

The point on the celestial sphere which is directly overhead is called the *zenith*; the point diametrically opposite the zenith is called the *nadir*. In Figure 41, ZA is the diameter of the celestial sphere which joins zenith and nadir. Observers at different positions on the earth will not have the same zenith. To fix the position of the zenith for a given point a on the earth, draw the radius of the earth,

Oa, and extend it to intersect the celestial sphere in point Z. Midway between the zenith and nadir is a great circle called the *celestial horizon*, NS in Figure 41.

If one were to move northward, the pole and zenith would become nearer together, that is, the arc PZ in Figure 41 would decrease. The angle between the equator and horizon, or arc ES, would also decrease. At the north pole, P would coincide with Z, and equator

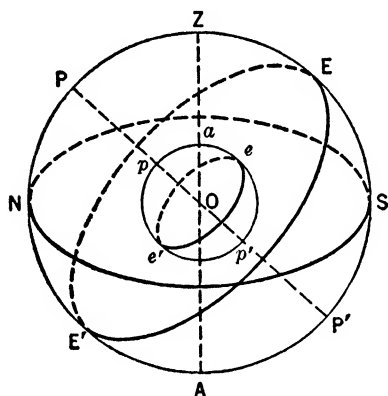


FIGURE 41.

and horizon would also coincide. On the earth's equator, P and N would coincide, and the celestial equator would be perpendicular to the horizon. For southern latitudes the north pole, P, would be below the horizon and the south pole, P', would be elevated above it.

The arc *ea* on the earth is the observer's latitude. The arc *EZ*, measured from the celestial equator to the zenith, is equal to the observer's latitude.

Problems

1. Show that the angle of elevation of the pole, arc NP in Figure 41, is the observer's latitude.
2. Draw figures like Figure 41 for latitudes of 15° N. and 75° N. Indicate the poles, celestial equator, zenith, and celestial horizon.
3. Repeat problem 2 for latitudes of 15° S. and 75° S.
4. Make a drawing of the celestial sphere in which the zenith and north pole coincide, that is, for latitude 90° N.
5. Draw a figure showing how the celestial sphere appears to an observer on the equator.

Co-ordinates on the Celestial Sphere; Right Ascension and Declination. The stars appear to have fixed positions on the celestial sphere, just as cities on the earth have a fixed latitude and longitude. The planets, on the other hand, appear to move among the stars or to change their co-ordinates, just as a moving ship on the earth changes its latitude and longitude. In fact, the name *planet* means wanderer. The moon may be seen to change its position among the stars. Observations of the moon on successive nights will show that

its angular distances from the stars along its path change rapidly. If the sun and stars could be observed at the same time, the sun would also appear to change its position relative to the stars.

If the changing position of the sun were plotted on the celestial sphere, its course would be found to be a great circle. This great circle is called the *ecliptic*, and makes an angle of $23^{\circ}27'$ with the equator. The sun moves eastward along the ecliptic and makes a complete journey around it, in one solar year or 365.2422 days. This annual movement of the sun around the ecliptic is, of course, distinct from its daily rising and setting. The sun has two motions; each day it moves from east to west across the sky as the celestial sphere makes a rotation, each year it completes one circuit of the ecliptic. The equator and the ecliptic intersect in two points, A and B, in Figure 42. The sun crosses the equator at point A, in a north-bound direction, about March 21; it crosses the equator again at B, in a south-bound direction, about September 21. Point A is called the *first point of Aries* or the *vernal equinox*. Point B is the autumnal equinox.

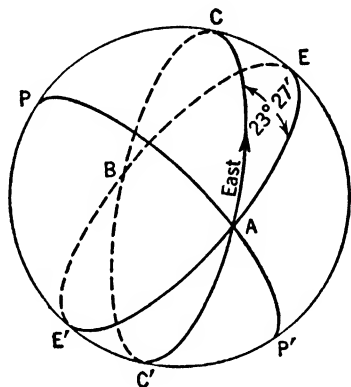


FIGURE 42.

Stars have a co-ordinate system which corresponds closely to latitude and longitude on the earth. The celestial co-ordinate corresponding to latitude is called *declination*. Like latitude, declination is measured north or south from the equator. The declination of all points on the equator is 0° ; the north pole has a declination of 90° N., or $+90^{\circ}$; the declination of the south pole is 90° S., or -90° . All points of a given declination lie on a small circle parallel to the equator.

The celestial co-ordinate corresponding to longitude is called *right ascension*. All the points on the half great circle or meridian PAP', which is drawn through the vernal equinox, have a right ascension of 0° (Figure 42). Right ascension, unlike longitude, is measured only eastward along the equator, from 0° to 360° . It is also expressed in hours and minutes.

$$24 \text{ hours, or } 24^{\text{h}} = 360^{\circ}$$

$$1 \text{ hour, or } 1^{\text{h}} = 15^{\circ}$$

$$1 \text{ second, or } 1^{\text{s}} = 15''$$

$$4 \text{ minutes, or } 4^{\text{m}} = 1^{\circ}$$

$$1 \text{ minute, or } 1^{\text{m}} = 15'$$

In Figure 43 the small circle BB' contains all the points of 60° N. declination. The meridian PCP' is drawn so that the arc $AC = 15^\circ$, or 1^h . The meridian PDP' is drawn so that the arc $ACED = 330^\circ$, or 22^h . The star S_1 has a declination 60° N. and a right ascension of 0^h . The co-ordinates of the star S_2 are 60° N. and 1^h . The star S_3 has the co-ordinates 60° N. and 22^h .

For some purposes a co-ordinate called *sidereal hour angle* is used in place of right ascension. Sidereal hour angle is measured *westward* along the equator, and is usually expressed in degrees and minutes, not in hours and minutes.

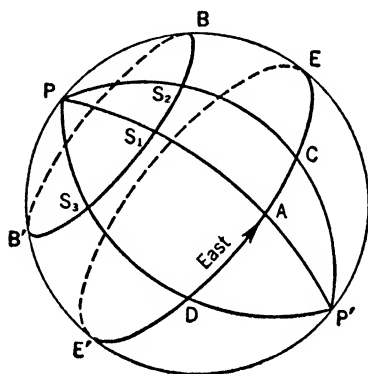


FIGURE 43.

One advantage of the co-ordinates right ascension and declination is that these co-ordinates are nearly constant for the stars. The right ascension and declination of the sun, moon, and planets change. Right ascension and declination are tabulated for the use of the navigator in the *American Nautical Almanac*, published yearly by the United States Naval Observatory.

Problems

1. The star Vega has a right ascension of $18^h 35^m$ and a declination of $+38^\circ 44'$. Draw a figure similar to Figure 43 and locate Vega by the intersection of its meridian and declination circle.

2. Repeat problem 1 for the following stars:

Star	Right Ascension		Declination	
	^h	^m	[°]	[']
Mizar	13	22	+ 55	13
Capella	5	12	+ 45	56
Pollux	7	42	+ 28	10
Regulus	10	05	+ 12	15
Altair	19	48	+ 8	43
Antares	16	26	- 26	18
Sirius	6	43	- 16	38
Rigel	5	12	- 8	16

3. Plot the positions of the stars given in problem 2 on a sheet of rectangular graph paper. Let the X axis represent the equator, and mark off the hours of right ascension at equal intervals along it; number the hours, 0 to 24, from *right to left*. Mark the right side of the map *west*, and the left side

east. Draw the meridians perpendicular to the equator, and use a scale so that 15° of declination has the same length as 1^h of right ascension.

4. Plot the stars of problem 2 on a map centered on the north pole. From the north pole as a center, draw equally spaced circles to represent the circles of declination. Draw radii from the pole, at intervals of 15° , to represent the meridians. Number the meridians counterclockwise from 0^h to 23^h .

Star Maps. Maps of the celestial sphere may be made in the same way as maps of the earth. In problems 3 and 4 of the last section, two kinds of star maps were made. One had the north pole as center; on the other the meridians and circles of declination were represented by straight lines. The same difficulties occur in making star maps as in making maps of the earth. It is inevitable that distances and directions are distorted. In the map of problem 3, for example, the lines representing the circles of declination are too long. In the map of problem 4, the distances along the meridians are correct, but the distances along the circles of declination are not.

In star maps of the type constructed in problem 3, the *right* side of the map is the west side. This map may be placed in its proper position by facing south and holding it over the head; this makes the right ascension lines point to the poles, and brings the east and west sides of the map in their proper position. The map of problem 4 should also be held over the head.

In orienting a star map, it is necessary to determine which one of the right ascension lines coincides with the observer's meridian. At some time during the twenty-four hours of the day each one of the right ascension lines coincides with the celestial meridian of any given place. The easiest method of finding the right ascension of the meridian is by using a small star map known as a *planisphere*. The planisphere is similar to the polar map of problem 4, but it is arranged so that the time of day may be set opposite the date. This determines the right ascension of the meridian mechanically.

An approximate method for finding the right ascension of the observer's meridian is as follows:

Multiply the number of months since September 21 by two and add to the number of hours since midnight.

Example 1. Find the right ascension of the meridian for 10 : 00 P.M. local time on August 11. 10 : 00 P.M. = 22 hours since midnight.

Sept. 21 to Aug. 11 = 10.7 months.

$10.7 \times 2 + 22 = 43.4$ hours.

Since 43.4 is more than 24, the right ascension of the meridian is $43.4 - 24 = 19.4$ hours approximately.

Example 2. Find the right ascension of the meridian for 4 : 00 A.M. local time on December 31.

4 : 00 A.M. = 4 hours since midnight.

Sept. 21 to Dec. 31 = 3.3 months.

$3.3 \times 2 + 4 = 10.6$ hours approximately.

This method is approximate only but is sufficiently accurate for orienting a star map. In using this rule, standard time should be used and not daylight saving or war time.

Problems

1. Find the approximate right ascension of the meridian for 9 : 00 P.M. local time for the first of each month.

2. Find the approximate right ascension of the meridian for the fifteenth of each month at 3 : 00 A.M. local time.

3. Plot the positions of the moon for January, 1943, on a star map. Use the moon's right ascension and declination for zero hours (midnight) Greenwich time for each day of the month. This data is given in the 1943 *Nautical Almanac* on pages 40–47. Connect the points with a smooth curve to show the moon's path among the stars.

4. Plot the position of the planet Venus for 1943. Use the right ascension and declination for the first and fifteenth of each month. (*Nautical Almanac*, pages 138–141.)

5. Repeat problem 5 for the planet Mars. (*Nautical Almanac*, pages 142–145.)

Declination and Hour Angle; the Second Co-ordinate System.

The co-ordinates right ascension and declination have two advantages.

They are independent of the place of observation, and they are nearly constant for the stars. On the other hand, the hours of right ascension are not directly connected with hours of time. For some problems it is convenient to use a co-ordinate called *hour angle* in place of right ascension.

Figure 44 represents the celestial sphere; P and P' are the poles, EE' the equator, NS the celestial horizon, and Z the zenith. At some time each day any given star

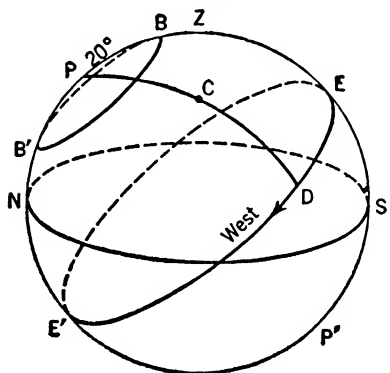


FIGURE 44.

crosses the local meridian PZP'. During the day the star will be carried around the pole in a circle. For example, if the star has a

declination of 70° N., it will describe a circle, BB' in Figure 44, which is 20° from the pole.

When the star crosses the arc of the meridian which contains the zenith, it is said to be making an *upper transit*. B, in Figure 44, shows the star at upper transit. When the star crosses PNP' , it is said to be making a *lower transit*. B' shows the star at lower transit. For all localities in the United States, both the upper and lower transits of a star of declination 70° N. are above the horizon NS. For certain stars only the upper transit is above the horizon. For other stars, both upper and lower transits are below the horizon. The time between two successive upper transits of a star is about four minutes less than twenty-four hours.

The angle between the meridian of the place of observation and the meridian through the star is called the *star's hour angle*. The hour angle of a star at upper transit is 0° , and at lower transit its hour angle is 180° . When the hour angle of the sun is 0° , it is noon by the sun or *local apparent noon*. Unlike right ascension, the hour angle is measured westward. Suppose a star is at point C, west of the meridian, in Figure 44; its hour angle is angle EPD, or the arc ED along the equator. When a star is 15° east of the meridian, its hour angle is 345° . Stars may be located on the surface of the celestial sphere by hour angle and declination. The hour angle, of course, varies with time; hence this co-ordinate system fixes the position of a star for a given time only.

Problems

In each of the following problems draw a figure to represent the celestial sphere. Locate the star from the given co-ordinates. Mark off the hour angle on the equator and the declination on a meridian.

	<i>Hour Angle</i>	<i>Declination</i>
1.	0°	$+ 10^\circ$
2.	330°	$- 20^\circ$
3.	180°	0°
4.	135°	$+ 30^\circ$
5.	270°	$+ 60^\circ$
6.	210°	$+ 20^\circ$
7.	45°	$- 15^\circ$

Hour Angle and Time. The *Greenwich hour angle* of a star is its hour angle measured from the meridian of Greenwich. The *American Nautical Almanac* and the *American Air Almanac* give the Green-

wich hour angle (G. H. A.) of the sun, the moon, Venus, Mars, Jupiter, Saturn, and 55 of the brighter stars. The G. H. A. is tabulated for frequent intervals of time, and special tables are given to aid in interpolation.

The sun's hour angle is a natural measure of time, one hour of time corresponding to a change of 15° in the hour angle. Noon by the sun, or local apparent noon, is the time when the sun reaches its greatest angle of elevation and has an hour angle of 0° . Suppose one observed the sun on the meridian of Greenwich and noted the Greenwich standard time at which the sun reached its greatest altitude above the horizon. The sun would not reach its highest point at exactly twelve noon by the Greenwich clock and the interval between two successive noons by the sun would not be exactly 24 clock hours. For example, on April 1, 1943, the sun at Greenwich reaches its highest altitude at 12 : 04 : 07.5 P.M.; on April 2, 1943, the highest altitude occurs at 12 : 03 : 49.5. The irregularity of the sun's motion is due to two causes; the sun moves along the ecliptic instead of on the equator; and its daily motion eastward along the ecliptic is not the same for all times of the year.

To explain the standard time in ordinary use, it is necessary to invent a "fictitious sun," which is called the *mean sun*. The mean sun is assumed to travel around the equator at a constant rate equal to the average rate of the real sun. The standard time of any meridian corresponds to the hour angle of the mean sun measured from that meridian. Thus:

<i>Hour Angle</i>	<i>Time</i>
180°	0 ^h Midnight
270°	6 ^h 6 : 00 A.M.
360° or 0°	12 ^h Noon
90°	18 ^h 6 : 00 P.M.
180°	24 ^h Midnight

Standard time may be based on any meridian. The standard times used in the United States and their meridians are: Eastern, 75° W.; Central, 90° W.; Mountain, 105° W.; and Pacific, 120° W. The War Times now in use and their meridians are Eastern, 60° W.; Central, 75° W.; Mountain, 90° W.; Pacific, 105° W. Data in the *Nautical Almanac* and *Air Almanac* are tabulated for *Greenwich Civil Time* (G. C. T.), which is the standard time of the 0° meridian. In using one of these almanacs, it is necessary to find the G. C. T. which corresponds to the standard time of some other meridian.

The system of time keeping used in the United States Navy extends the principle of standard time to the entire globe. The Navy uses twenty-four time zones as shown in Figure 45. The ship time in any zone is the standard time of the mid-meridian of the zone. The numbers in each zone show the number of hours to be added to zone time to obtain G. C. T. In zone -7 , between longitude $97^{\circ} 30' \text{ E.}$ and $112^{\circ} 30' \text{ E.}$, a ship keeps the standard time of the meridian of 105° E. To change ship time in this zone to G. C. T. subtract 7 hours.

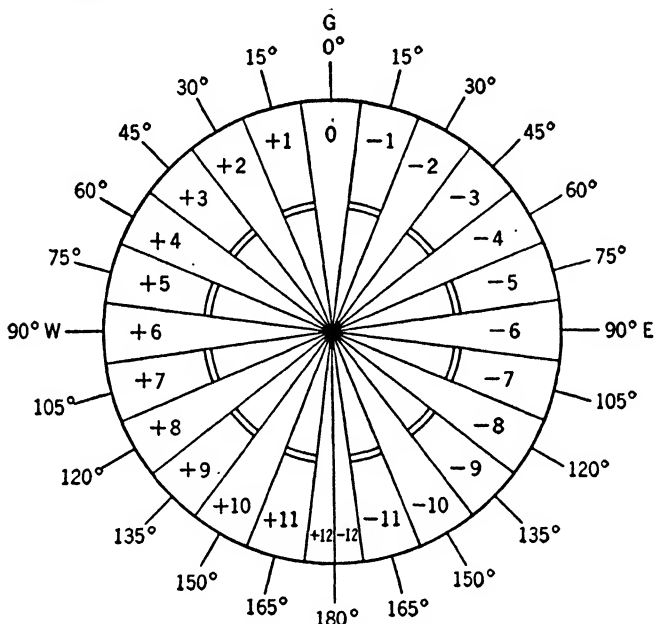


FIGURE 45. (After Dutton.)

Example 1. Find the G. C. T. and date corresponding to 9 : 30 P.M. E. W. T., January 27, 1943.

E. W. T.	9 : 30 P.M.	
Zone Time	21 ^h 30 ^m	January 27
Zone	+ 4 ^h	
	<hr/>	
	25 ^h 30 ^m	
G. C. T.	1 ^h 30 ^m	January 28

Notice in this example that the Greenwich date is later than the E. W. T. date.

Example 2. Find the G. C. T. and date corresponding to $8^{\text{h}} 00^{\text{m}}$ Feb. 3, 1943, Zone -10 .

Zone Time	$8^{\text{h}} 00^{\text{m}}$	February 3
Zone	$- 10^{\text{h}} 00^{\text{m}}$	
G. C. T.	$22^{\text{h}} 00^{\text{m}}$	February 2

In this example, the Greenwich date is earlier than the local date, since $8^{\text{h}} 00^{\text{m}} - 10^{\text{h}} 00^{\text{m}} = -2^{\text{h}} 00^{\text{m}}$ or $22^{\text{h}} 00^{\text{m}}$ of the previous day. In other examples the local date and the Greenwich date may be the same.

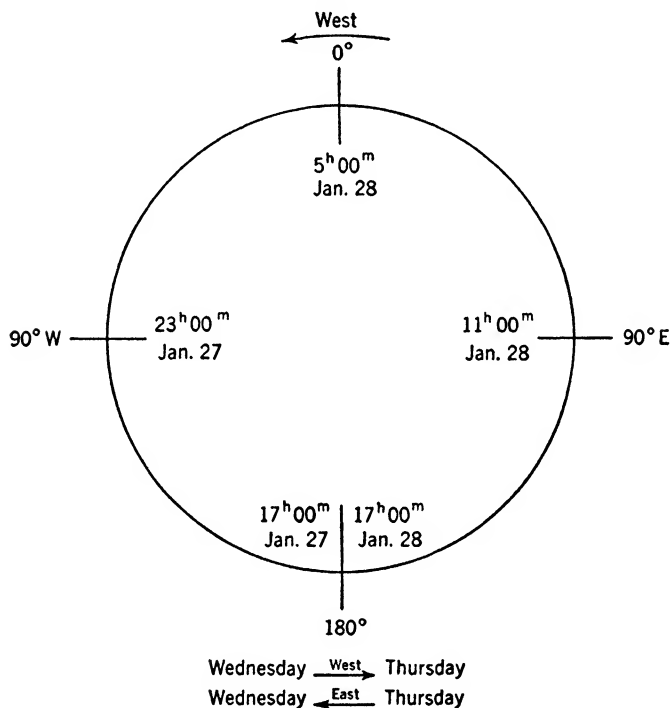


FIGURE 46.

The 12-hour zone in Figure 45 is divided into two parts, $+12$ and -12 . Between $172^{\circ} 30' \text{ W.}$ and 180° , the local time is 12 hours slower than G. C. T.; between $172^{\circ} 30' \text{ E.}$ and 180° the local time is twelve hours faster than G. C. T. The date must be changed when the meridian of 180° is crossed. The reason for this change of date is shown in Figure 46. Suppose it is $5^{\text{h}} 00^{\text{m}}$ on January 28 at Greenwich; west of Greenwich the local time is earlier, and hence at 180° the local time would be $17^{\text{h}} 00^{\text{m}}$, January 27. East of

Greenwich the local time is ahead of G. C. T., and hence at 180° it would be $17^h 00^m$ January 28.

Problems

1. Find the G. C. T. and date corresponding to the following local times and dates.

- (a) $11 : 00$ P.M. E. W. T., Jan. 27
- (b) $10 : 00$ A.M. P. S. T., Jan. 29
- (c) $12 : 00$ M. C. S. T., Feb. 14
- (d) $4 : 00$ A.M. E. S. T., Mar. 22
- (e) $7 : 30$ P.M. C. W. T., Mar. 25
- (f) $5 : 30$ A.M. P. W. T., April 3
- (g) $9 : 30$ P.M. M. S. T., April 17
- (h) $11 : 30$ A.M. M. W. T., April 21

2. Find the G. C. T. and date corresponding to the following:

- (a) $10^h 00^m$, Jan. 3, Zone + 4
- (b) $2^h 00^m$, Feb. 19, Zone - 6
- (c) $2^h 00^m$, Mar. 3, Zone + 12
- (d) $3^h 00^m$, April 16, Zone - 10
- (e) $17^h 00^m$, May 23, Zone - 12
- (f) $17^h 00^m$, May 29, Zone + 8
- (g) $15^h 00^m$, June 1, Zone - 9

Altitude and Azimuth; the Third Co-ordinate System. The first co-ordinate system considered was right ascension and declination; the second was hour angle and declination. The system of right ascension and declination is independent of the place of observation, so that these co-ordinates may be used in making star maps. The hour angle was found to be a means of measuring time. The co-ordinates, right ascension, declination, and hour angle cannot be measured directly by an observer.

The co-ordinates which an observer might determine for himself are the angle of elevation of a star and its bearing or direction. In astronomy and navigation the angle of elevation of a star is called its *altitude*. The direction or bearing of the star is called its *azimuth*. Azimuth, like bearing on the earth, is measured from 0° to 360° — north, 0° ; east, 90° ; south, 180° ; west, 270° .

Figure 47 represents the celestial sphere; PZS, the meridian; Z, the zenith; and N, E, S, and W, the north, east, south, and west points of the horizon. Altitude is measured from 0° on the horizon to 90° at the zenith. A star below the horizon has a negative altitude. Stars of equal altitude lie on a small circle, which is parallel

to the horizon. AA' in Figure 47 contains all points which have an altitude of 60° . The points on the arc ZPN are due north or have

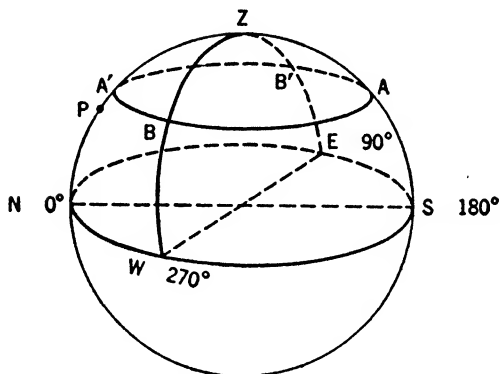


FIGURE 47.

an azimuth of 0° . If great circle arcs are drawn from Z to E , S , and W , then all points on ZE have an azimuth of 90° , all points on ZS an azimuth of 180° , etc. The azimuth may be regarded either as an angle at Z or as an arc of the horizon, measured from N in an eastward direction.

In Figure 47 the altitude and azimuth of the four points A' , B' , A , and B are as follows:

<i>Point</i>	<i>Altitude</i>	<i>Azimuth</i>
A'	60° , arc NA'	0°
B'	60° , arc EB'	90° , arc NE
A	60° , arc SA	180° , arc NES
B	60° , arc WB	270° , arc $NESW$

Problems

On a separate sheet of paper complete the following statements:

1. The distance of a star from the equator is -----; from the celestial horizon is -----.

2. Declination is 0° on the -----; $+90^\circ$ at the -----; -90° at the ----- . Altitude is 0° on the -----; $+90^\circ$ at the -----; -90° at the -----.

3. Stars of equal altitude lie on a ----- parallel to the ----- . Stars of equal declination lie on a ----- parallel to the -----.

4. A star is at upper transit when it crosses the great circle arc containing the poles and the ----- . A star is at lower transit when it crosses the great circle arc containing the poles and the -----.

5. The local hour angle of a star is the angle between the local meridian and the ----- . At upper transit the hour angle = ----- . At lower transit the hour angle = -----.

6. Stars that have hour angles between 0° and 180° are ----- of the local meridian. Stars that have hour angles between 180° and 360° are ----- of the local meridian.

7. Right ascension is the angle between the meridian through the star, and the meridian through the _____. It is measured _____ from 0° to 360° , or from _____ to _____.

8. Azimuth is the angle between the great circle through the zenith and north pole, and the great circle through the ----- . Azimuth is measured from 0° to ----- through ----- .

9. The right ascension of the sun changes 360° or 24^h in one
The right ascension of the moon changes 360° or 24^h in about days.

10. Tell what co-ordinates of a star vary with time? with the place of observation?

11. What co-ordinates are used for star maps? Why are these co-ordinates used?

12. Why do star maps not show the sun, moon, and planets?

The Astronomic Triangle. Most of the problems in celestial navigation are concerned with solving a spherical triangle. This is the triangle formed on the celestial sphere by the pole, the observer's zenith, and a given star. In Figure 48 the triangle PZS_i is formed by the north pole, P, the observer's zenith, Z, and a star, S_i. The great circle EE' is the equator, and NS is the horizon. The sides of this triangle are easily related to various co-ordinates.

FIGURE 48.

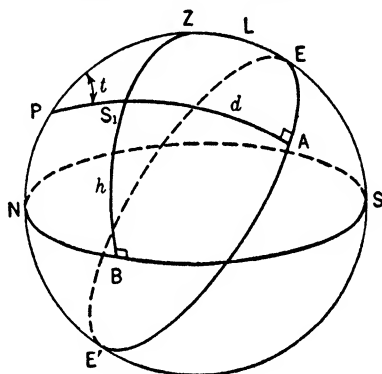


FIGURE 48.

EZ = observer's latitude, **L** **PZ** = $90^\circ - L$

$$\text{AS}_1 = \text{star's declination, } d \quad \text{PS}_1 = 90^\circ - d$$

$$\text{BS}_1 = \text{star's altitude, } h \qquad \text{S}_1\text{Z} = 90^\circ - h$$

South latitudes and declinations are considered negative, so the expressions for sides of the triangle are true for all latitudes and declinations.

In Figure 48 the star is west of the meridian, and the angle ZPS_1 = the star's local hour angle. When a star is east of the meridian, the angle ZPS_1 would be 360 minus the local hour angle. If angle P is denoted by t and the local hour angle by L. H. A., then,

angle P = t = L. H. A., or $360 - \text{L. H. A.}$

In this figure the azimuth is the arc NSB of the horizon or the corresponding angle at Z; and angle PZS₁, or angle Z in this triangle, is

360° minus the azimuth. When the star is east of the meridian, then angle Z in the triangle equals the azimuth. Hence, if the azimuth is denoted by Z_n , then

$$\text{angle } Z = Z_n, \text{ or } 360 - Z_n$$

The relations between the sides and angles of the astronomic triangle and the celestial co-ordinates may be summarized:

$$PZ = 90^\circ - \text{latitude} = 90^\circ - L, \text{ south latitudes negative}$$

$$ZS_1 = 90^\circ - \text{altitude} = 90^\circ - h$$

$$PS_1 = 90^\circ - \text{declination} = 90^\circ - d, \text{ south declinations negative}$$

For a star west of the meridian:

$$\text{angle } ZPS_1 = t = \text{local hour angle} = \text{L. H. A.}$$

$$\text{angle } PZS_1 = 360 - \text{azimuth} = 360^\circ - Z_n$$

For a star east of the meridian:

$$\text{angle } ZPS_1 = t = 360 - \text{local hour angle} = 360^\circ - \text{L. H. A.}$$

$$\text{angle } PZS_1 = \text{azimuth} = Z_n$$

Angle ZS_1P is not involved in the problems of celestial navigation.

The astronomic triangle corresponds exactly to a triangle on the earth, formed by the pole of the earth, the point of observation, and the point directly underneath the star. There are many problems in which it is required to solve the astronomic triangle. We shall learn in the next chapter that the main problem in celestial navigation is to compute the altitude and azimuth from the latitude, declination, and hour angle corresponding to a given time and place. This problem may be solved by the haversine and sine formulas of Chapter V.

Declination and G. H. A. are given in the *Nautical Almanac* and *Air Almanac* at frequent intervals of G. C. T. From the G. H. A. one may find either the L. H. A. or $360 - \text{L. H. A.}$, that is, angle t in the triangle.

Example 1. Using the *Nautical Almanac*, find the values of angle t and the declinations for the stars Aldebaran, Capella, and Regulus at $40^\circ 37' \text{ N.}$, $92^\circ 36' \text{ W.}$ for February 25, 1943, at 10 : 32 : 45 P.M. C. S. T.

C. S. T.	10 : 32 : 45 P.M.
Zone time	$22^{\text{h}} 32^{\text{m}} 45^{\text{s}}$
Zone	+ 6^{h}
G. C. T.	$4^{\text{h}} 32^{\text{m}} 45^{\text{s}}$, February 26

The G. H. A. and declinations are found in the 1943 *Nautical Almanac* under "Stars, February, 1943," pages 170-173. The stars are listed in the

order of right ascension. Under the name of each star is found its right ascension, its declination, and the time it transits the meridian of Greenwich on the first of the month. For each star the G. H. A. is given for 0^h G. C. T. for each day of the month. The numbers in parentheses below refer to pages in the *Nautical Almanac*.

	<i>Aldebaran</i>	<i>Capella</i>	<i>Regulus</i>
Declination	16° 24' (170)	+ 45° 56' (170)	+ 12° 15' (171)
G. H. A. 0 ^h Feb. 26	86° 48'.4 (170)	76° 50'.9 (171)	3° 37'.8 (171)
Corr. for 4 ^h 32 ^m	68° 11'.2 (214)	68° 11'.2 (214)	68° 11'.2 (214)
Corr. for 45 ^s	11'.3 (214)	11'.3 (214)	11'.3 (214)
G. H. A. for 4 ^h 32 ^m 45 ^s	155° 11'	145° 13'	72° 00'
Longitude	92° 36'	92° 36'	92° 36'
	62° 35' west	52° 37' west	20° 36' east

The stars Aldebaran and Capella are west of the meridian of 92° 36'. The answer of 20° 36' for Regulus is of course 360° - L. H. A., since the star is east of the meridian. In order to avoid errors in finding the angle t , diagrams as shown in Figure 49 should always be drawn.

The three diagrams in Figure 49 show how the longitude and the G. H. A. are combined to find angle t . These diagrams represent the angles plotted on the equator as they would appear if viewed from the south. In each case draw the vertical diameter Mpm; Mp being the local meridian of 92° 36' W. Draw the line Gpg making angle MpG = 92° 36', so that pG represents the meridian of Greenwich. The star's G. H. A. is laid off to the westward of pG. Angle t in each case is the angle ★ pM. It is obvious from the figures when angle t is west and when it is east of pM.

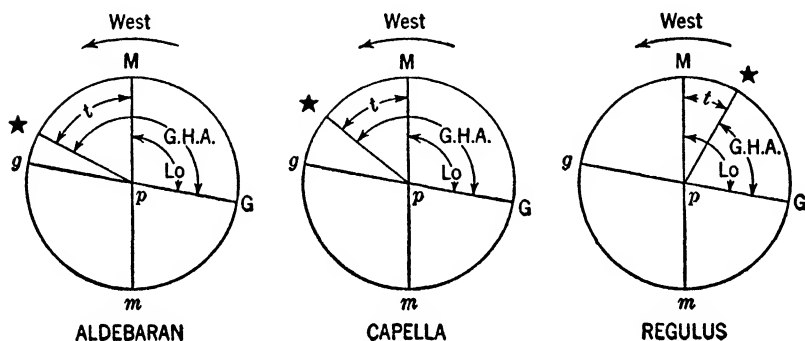


FIGURE 49.

Example 2. Find angle t and the declination of the sun for $15^{\circ} 23' \text{ S.}$, $44^{\circ} 48' \text{ E.}$ at $15^{\text{h}} 45^{\text{m}} 37^{\text{s}}$ zone time on March 11, 1943 (see Figure 50).

Zone time $15^{\text{h}} 45^{\text{m}} 37^{\text{s}}$
 Zone $- 3^{\text{h}}$
 G. C. T. $12^{\text{h}} 45^{\text{m}} 37^{\text{s}}$, March 11

"The Sun for March 11" is page 10 in the *Nautical Almanac*, and the "Correction Table for the G. H. A." is page 12. The sun's declination is $- 3^{\circ} 56'$; found by inspection on page 10 between the tabulated values for 12^{h} and 14^{h} .

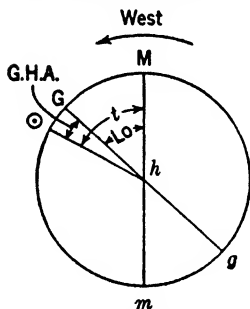


FIGURE 50.

G. H. A. for 12^{h}	357° 25.5	(10)
Corr. for 45^{m}	11° 15.0	(12)
Corr. for 37^{s}	09.3	(12)
	<hr/> 368° 50'	
G. H. A.	8° 50'	
Longitude	<hr/> 44° 48'	
	<hr/> 53° 38' west	

Examples of finding G. H. A.'s for the moon and the planets are given in the 1943 *Nautical Almanac* on pages 304–305. In the *Air Almanac* the G. H. A. and declination are tabulated at ten-minute intervals for the sun, moon, and planets. The G. H. A. of a star is found from the star's sidereal hour angle and the tabulated value of the G. H. A. of the vernal equinox, or first point of Aries. The G. H. A. of the first point of Aries is indicated by the symbol G. H. A. φ .

Example 3. Find the G. H. A. of the Sun, Mars, and Moon for $4^{\text{h}} 27^{\text{m}} 32^{\text{s}}$ G. C. T. November 13, 1942, using the *Air Almanac* for September–December, 1942.

	<i>Sun</i>	<i>Mars</i>	<i>Moon</i>
G. H. A. for $4^{\text{h}} 20^{\text{m}}$ (633)	248° 56'	261° 18'	182° 51'
Corr. for $7^{\text{m}} 32^{\text{s}}$ (inside front cover)	1° 53'	1° 53'	1° 49'
Additional corr. for Moon (633)			– 1'
G. H. A.	<hr/> 250° 49'	<hr/> 263° 11'	<hr/> 184° 39'
Declination (633)	– 17° 47'	– 13° 41'	– 18° 30'

Example 4. Find the G. H. A. and declination of the stars Antares and Canopus for $4^{\text{h}} 27^{\text{m}} 32^{\text{s}}$ G. C. T., November 13, 1942 (see Figure 51).

	<i>Antares</i>	<i>Canopus</i>
G. H. A. φ for $4^{\text{h}} 20^{\text{m}}$ (633)	116° 39'	116° 39'
Corr. for $7^{\text{m}} 32^{\text{s}}$ (inside front cover)	1° 53'	1° 53'
Sidereal H. A. of star (inside back cover)	<hr/> 113° 32'	<hr/> 264° 20'
G. H. A. of star	<hr/> 232° 04'	<hr/> 22° 52'
Declination (inside back cover)	– 26° 18'	– 52° 40'

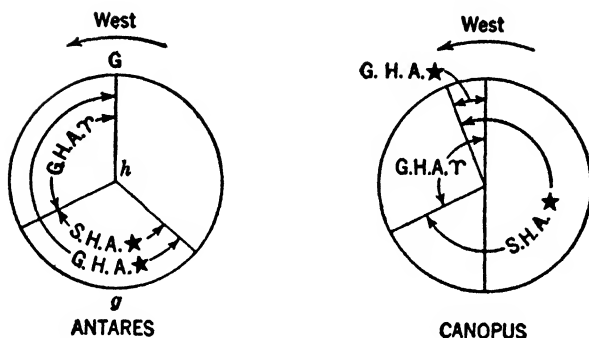


FIGURE 51.

Problems

- Find the values of angle t , which correspond to G. H. A.'s of $10^{\circ} 15'$, $275^{\circ} 49'$, $110^{\circ} 27'$, $235^{\circ} 29'$, $342^{\circ} 25'$, for the following longitudes:
(a) $15^{\circ} 18' \text{ W.}$; (b) $135^{\circ} 42' \text{ E.}$; (c) $175^{\circ} 43' \text{ W.}$; (d) $96^{\circ} 18' \text{ E.}$
- Find the values of angle t , which correspond to G. H. A.'s of $317^{\circ} 42'$, $43^{\circ} 25'$, $286^{\circ} 24'$, $135^{\circ} 49'$, $93^{\circ} 23'$, for longitudes:
(a) $74^{\circ} 00' \text{ W.}$; (b) $74^{\circ} 00' \text{ E.}$; (c) $145^{\circ} 33' \text{ E.}$; (d) $145^{\circ} 33' \text{ W.}$
- Using the *Nautical Almanac* for 1943, find the G. H. A., angle t , and declination of the stars Dubhe, Regulus, and Spica:
(a) For a point in longitude $110^{\circ} 23' \text{ E.}$ at $3^{\text{h}} 40^{\text{m}} 27^{\text{s}}$ zone time, January 15 (pages 166–169).
(b) For a point in longitude $127^{\circ} 49' \text{ W.}$ at $21^{\text{h}} 20^{\text{m}} 33^{\text{s}}$ zone time, April 14 (pages 178–181).
- Using the *Nautical Almanac* for 1943, find the G. H. A., angle t , and declination of the stars Vega, Deneb, and Fomalhaut:
(a) For a point in longitude $96^{\circ} 43' \text{ W.}$ at $4^{\text{h}} 15^{\text{m}} 55^{\text{s}}$ zone time, July 5.
(b) For a point in longitude $89^{\circ} 33' \text{ E.}$ at $22^{\text{h}} 04^{\text{m}} 19^{\text{s}}$ zone time, October 6.

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The Star Explorer, Junior Astronomy Club, Hayden Planetarium, New York.

Illyne's Star Chart, Weems System of Navigation, Annapolis, Maryland.

Outline Star Charts of the Northern and Southern Hemispheres, Rand McNally & Co., Chicago.

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Nautical Astronomy:

Dutton, Chapter VIII; Bowditch, Chapter VII; Lyon, Chapter X.

Time:

The *Nautical Almanac*; the *Air Almanac*; Dutton, Chapter XII; Bowditch, Chapter IX; Lyon, Chapter X.

(The *American Nautical Almanac* is published for each year in advance by the United States Naval Observatory. The *American Air Almanac* also is published by the United States Naval Observatory; each volume covers a period of four months.)

General References:

Bartky, *Highlights of Astronomy*, University of Chicago Press.

Bernhard, Bennett, Rice, *New Handbook of the Heavens*, McGraw Hill, New York.

Barton and Joseph, *Starcraft*, McGraw Hill, New York.

CHAPTER VII

CELESTIAL NAVIGATION; THE SUMNER LINE

From time immemorial observations of the stars have been used to determine directions and to aid in finding position on the earth. History records many early navigators who used celestial navigation. The methods and instruments used in modern celestial navigation are, however, of comparatively recent development. Columbus relied principally on dead reckoning but used the North Star to tell the time at night, to check the variation of the compass, and to find his latitude. The early Polynesian navigators sailed for thousands of miles across the Pacific and fixed their courses by the stars.

Kupé, the first Polynesian discoverer of New Zealand, who is said to have lived about the thirteenth or fourteenth century, gave to succeeding expeditions from Tahiti very particular sailing directions, that they should steer always a little to the right of the place where the Sun, Moon, and Venus all set at the beginning of the hot season.

T. R. ST. JOHNSTON in *Islanders of the Pacific*

The early Hawaiians sailed to Tahiti, a distance of more than 2300 nautical miles. The return voyage was directed to the eastward of Hawaii, and the proper time to turn west was determined by observing the Pole Star until it had an altitude equal to the latitude of Hawaii.

The development of modern precise navigation is due in no small part to the work of the self-taught American, Nathaniel Bowditch. In 1795, Bowditch went to sea, as supercargo on a vessel engaged in the China trade. He devoted the leisure which this position gave him to the study of navigation and the correction of the existing manual of navigation. *The American Practical Navigator*, published at intervals by the United States Hydrographic Office, bears on its title page the statement, "originally by Nathaniel Bowditch, LL.D.," and is universally known simply as *Bowditch*.

Before 1761 the determination of longitude at sea was very difficult because time pieces were not sufficiently accurate to be reliable on a long voyage. The British Government offered large rewards for an accurate clock, or *chronometer*, suited to the purposes of navigation.

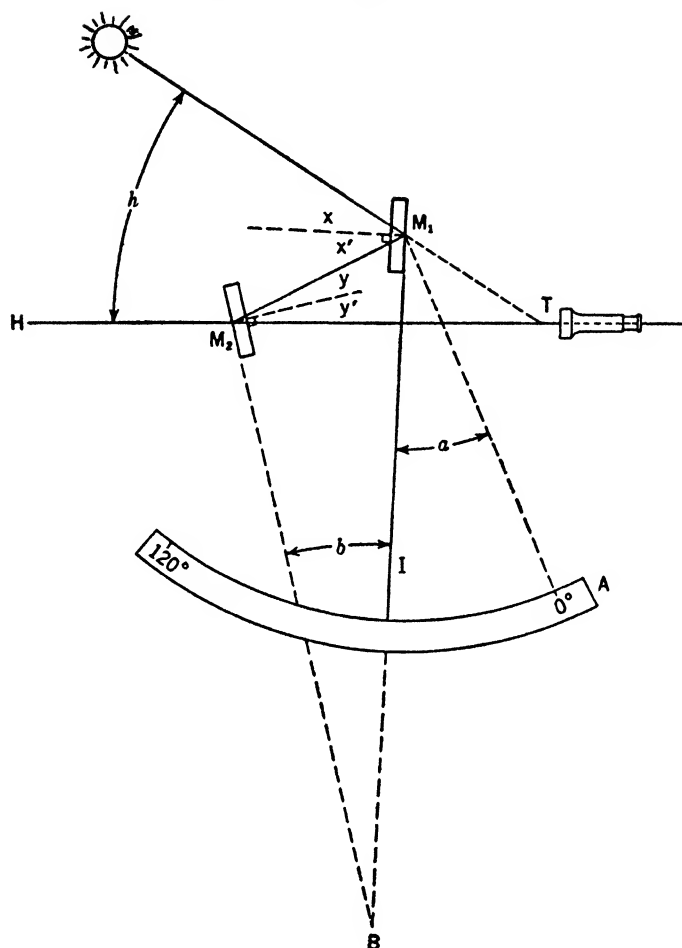


FIGURE 52.

The first chronometer suitable for finding the longitude at sea was invented by John Harrison in 1761. Harrison received a reward of £20,000 from the British Government.

The altitudes of the sun and stars are measured by an instrument called the *sextant*. The sextant measures the angle subtended at the observer's eye by distant objects. The name sextant is derived from the length of the arc on the instrument, 60° . The *octant* is similar to the sextant except that it has an arc of 45° . The principle of measuring angles by reflection, which is used in the sextant, was first applied by the Englishman, Hadley, and an American, Godfrey,

about 1730. The sextant or octant replaced an earlier instrument, the *astrolabe*, which was used for measuring altitudes. The name *astrolabe* means "to take a star." Finding the altitude of the sun or a star is called "shooting" the sun or star.

Measurement and Correction of Observed Altitudes. The essential parts of the *marine sextant* are shown in Figure 52:

A is the arc, or limb, graduated from 0° to 120° .

I is the movable index arm, which is mounted to swing about the center of the arc, A.

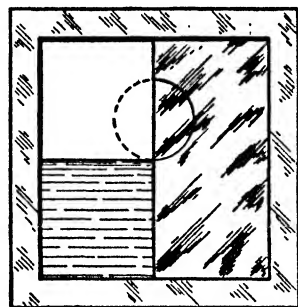
M_1 is a mirror, called the index glass; it is mounted on the index arm and is perpendicular to the plane of the arc.

M_2 is the horizon glass. Its plane is also perpendicular to the arc, A. The right half of the horizon glass is a mirror, the left half is clear glass.

T is a telescope directed to the horizon glass. The axis of the telescope is parallel to the plane of the arc.

Sextants are provided with dark glasses for use in observing the sun. To measure the sun's altitude, the sextant is held vertically, and the horizon, H, is viewed through the telescope in the line TH. The index arm is turned until the reflected image of the sun appears tangent to the horizon as shown in Figure 53. The sun's altitude above the horizon is angle h in Figure 52. The angle through which the index mirror has been turned is angle a . The arc is 60° in length, but the scale is doubled so that the angle read is $2a$. The principle of the sextant is that

$$\text{angle } h = 2 \text{ angle } a.$$



Clear Mirror

FIGURE 53.

When a sextant is in perfect adjustment, the line from M_1 to 0° on the arc, is parallel to the horizon glass; that is, the line

$$M_1 0^\circ \text{ is parallel to } M_2 B$$

and hence

$$\text{angle } a = \text{angle } b.$$

When light strikes a mirror and is reflected, the ray that strikes the mirror and the reflected ray make equal angles with the perpendicular to the mirror (*i.e.*, the angle of incidence equals the angle of reflection). In Figure 52

$$\text{angle } x = \text{angle } x' \qquad \text{angle } y = \text{angle } y'.$$

It follows from geometry, that h , the required altitude, equals $2a$, which is the angle read from the arc, for

$$\begin{aligned} \text{in triangle } TM_1M_2 \quad h &= (x + x') - (y + y') \\ &= 2x' - 2y \end{aligned}$$

$$\begin{aligned} \text{in triangle } M_1M_2B \quad b &= 180^\circ - (90^\circ + y) - (90^\circ - x') \\ &= x' - y \end{aligned}$$

$$\text{hence} \quad h = 2b$$

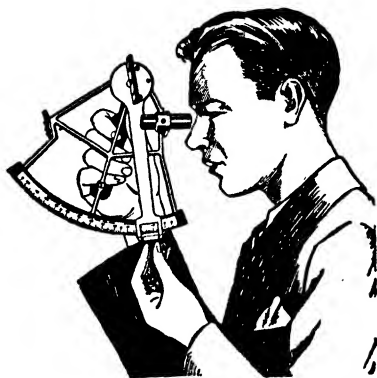
$$\text{Since} \quad b = a, \quad h = 2a$$

Sextant altitudes must be corrected for an *index error*. Index error is most accurately determined by making the direct and reflected images of a star coincide. If the angle reads zero, when the two images coincide, there is no index error; if the reading is 1° to the *left* of zero on the arc, then one degree must be *subtracted* from all altitudes. If the reading is 1° to the right, then one degree must be added to all altitudes. The navigator uses the following rule to tell when to add and subtract index error:

If it's on, it's off; if it's off, it's on.

Index error may also be determined by making the direct and reflected images of the horizon coincide.

On land, altitudes of the sun and stars may be observed with a



Courtesy Yoder Instruments

USING THE MARINE SEXTANT.

surveying transit, if the altitudes are not greater than 45° . Instrument companies supply sun glasses for use with a transit. When a transit is used to observe star altitudes, the cross wires must be illuminated. This may be done by directing the light of a small flashlight down the telescope, in such a way as not to obstruct the vision. The transit should first be focused on a distant object.

The marine sextant is used when a perfect sea horizon is available.

Bubble sextants have been developed which eliminate the necessity for sighting the sea horizon. They are particularly suited for aerial navigation.

A number of corrections must be applied to observed altitudes. These corrections are tabulated in the *Nautical Almanac* and the *Air Almanac* and are easy to apply in practice.

(1) *Index Error.* The index error of a sextant, which has just been discussed, must be determined frequently and applied to all altitudes measured with the sextant.

(2) *Dip or Height of Eye.* This is a correction applied only to altitudes measured from the visible sea horizon; it does not apply to altitudes measured with the bubble sextant or transit. The line of sight toward the horizon is below the horizontal, hence the angle observed is too large. Corrections for dip are always subtracted. Corrections for various heights of eye are given in Table C, page 295, in the *Nautical Almanac*, and on the back cover of the *Air Almanac*.

(3) *Semidiameter.* In Figure 52, the altitude of the sun's lower edge was observed. This altitude must be corrected to obtain the altitude of the sun's center. Corrections for semidiameter are combined with corrections for refraction and parallax. When a bubble sextant is used, the center of the sun or moon is brought into coincidence with the bubble, and there is no semidiameter correction. The lower edge of the sun or moon is called the *lower limb*.

(4) *Refraction and Parallax.* The "bending" or refraction of light by the earth's atmosphere causes the stars to appear higher than

TABLE 1

CORRECTIONS TO OBSERVED ALTITUDES FROM THE "NAUTICAL ALMANAC"

	<i>Marine Sextant</i> Always correct for index error. Correct for Dip, Table C, page 295.	<i>Bubble Sextant</i> Always correct for index error. No Dip Correc- tion.	<i>Transit</i> No Index Error. No Dip.
Sun's Lower Limb	Table A, page 295.	-----	Table A
Sun's Center	-----	Table E, page 298.	-----
Star or Planet	Table A or Table E	Table A or Table E	Table A or Table E
Moon's Lower Limb ¹	Table D, pages 296-297.	-----	Table D
Moon's Upper Limb ¹	Table D	-----	Table D
Moon's Center ¹	-----	Table F, page 299.	-----

¹ In correcting altitudes of the moon, always find the horizontal parallax for the date of observation. Horizontal parallax is given at the bottom of each page for the "Moon 1943," pages 40-131. Use the horizontal parallax to find the correction in Table D, E, or F.

they are. Theoretically, an altitude should be measured from the center of the earth, instead of on its surface. Parallax is the difference between an altitude measured on the earth's surface and the same altitude measured from the earth's center. Parallax is small except in the case of the moon. Corrections for refraction and parallax are combined in a single table. For the sun and moon, corrections for semidiameter, refraction, and parallax are combined.

Table 1 (page 81) shows what tables from the *Nautical Almanac* should be used for correcting the various altitudes measured with sextants or with a transit.

Corrections for refraction and dip are given on the back cover of the *Air Almanac*. The semidiameters of the sun and moon are given on the A.M. side of each daily sheet. Corrections for the moon's parallax are also given on the A.M. side of each daily sheet.

Example 1. The sun's lower limb and a star are observed with a marine sextant having an index error of $-02'$. The height of eye is 37 feet. The altitude of the sun's lower limb is $42^{\circ} 36'$, and the star's altitude is $37^{\circ} 42'$. Find the corrected altitudes.

Nautical Almanac

Sun's Lower Limb		Star	
Index	$-02'$	Index	$-02'$
Dip 37 feet, Table C	$-06'$	Dip	$-06'$
Table A	$+15'$	Table A	$-01'$
Correction	$+7'$	Correction	$-09'$
Observed Altitude	$42^{\circ} 36'$	Observed Altitude	$37^{\circ} 42'$
Obs. Alt. Corr.	$42^{\circ} 43'$	Obs. Alt. Corr.	$37^{\circ} 33'$

Air Almanac

Sun's Lower Limb		Star	
Index	$-02'$	Index	$-02'$
Dip, Back Cover	$-06'$	Dip, Back Cover	$-06'$
Refraction, Back Cover	$-01'$	Refraction, Back Cover	$-01'$
Semidiameter, Daily Sheet	$+16'$	Correction	$-09'$
Correction	$+7'$		

Example 2. The altitude of the moon's center, observed on Nov. 27, 1942, with a bubble sextant, is $66^{\circ} 18'$. The sextant has an index error of $+05'$. Find the corrected altitude.

In the *Nautical Almanac* find the horizontal parallax for Nov. 27 at bottom of page 99. Use this value to obtain the correction from table F. In the *Air Almanac* find the correction on the right side of page 661.

<i>Nautical Almanac</i>		<i>Air Almanac</i>	
Horizontal Parallax	54' (page 122)	Index	+ 05'
Index Error	+ 05'	Refraction, Back Cover	00'
Table F	+ 21'	Parallax (page 661)	+ 21'
Correction	+ 26'	Correction	26'
Observed Altitude	66° 18'	Obs. Altitude	66° 18'
Obs. Alt. Corr.	66° 44'	Obs. Alt. Corr.	66° 44'

Problems

Correct the following altitudes.

	<i>Instrument</i>	<i>Index Error</i>	<i>Obs. Alt.</i>
1. Star	Bubble Sextant	- 05'	29° 32'
2. Sun's Lower Limb	Transit	-----	35° 47'
3. Mars	Bubble Sextant	+ 03'	28° 13'
4. Jupiter	Bubble Sextant	- 08'	45° 53'
5. Star	Transit	-----	44° 18'
6. Sun's Lower Limb	Bubble Sextant	+ 07'	27° 19'

7. Star altitudes of 32° 37', 65° 19', and 29° 18' have been found with a marine sextant having an index error of - 05'. The height of eye is 60 feet.

8. Correct the following altitudes of the Moon.

	<i>Altitude</i>	<i>Date</i>	<i>Index Error</i>
Upper Limb	32° 19'	Jan. 13, 1943	0
Center	45° 15'	Feb. 27, 1943	+ 04'
Lower Limb	63° 47'	Mar. 11, 1943	- 03'
Center	23° 56'	June 17, 1943	+ 05'

Latitude by Meridian Altitude. The latitude of a place may be determined by measuring the altitude of a star or other celestial body when it crosses the meridian. North of latitude 23° 27' N., the sun is always south at local apparent noon (see local apparent noon, page 66). Between the tropics (that is, between 23° 27' N. and 23° 27' S.) the sun may be north or south, depending on the time of the year. In the southern hemisphere, south of latitude 23° 27' S., the sun will always be north at noon.

Example 1. A navigator finds the sun's corrected altitude to be 33° 44' when the sun is crossing the meridian to the south. The sun's declination is 25° 33' S. What is the latitude?

The latitude may be determined by constructing Figure 54. Draw the semicircle NZS, to represent the meridian; N is the north point, Z the zenith, and S the south point. Plot the sun on this figure by making angle h equal to the altitude, 33° 44'. The intersection E of the meridian and the equator is found by making angle d equal to the declination 25° 33' S. Since the

sun is south of the equator, the equator is north of the sun. The angle EOZ is the observer's latitude, L . For this figure:

$$L + d + h = 90^\circ$$

$$L = 90^\circ - (d + h)$$

$$L = 30^\circ 43' \text{ N.}$$

In Figure 54, the latitude is north, since the zenith is north of the equator.

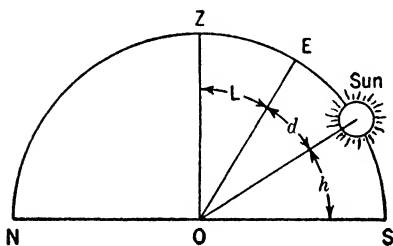


FIGURE 54.

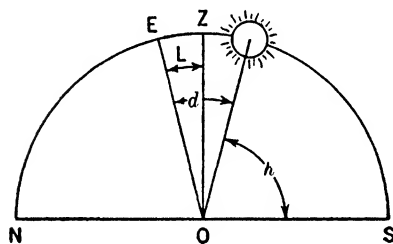


FIGURE 55.

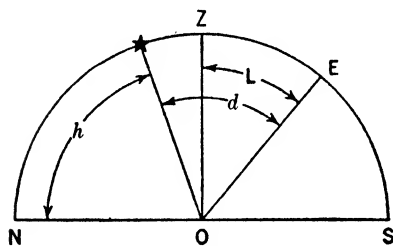


FIGURE 56.

The sun, in Figure 54, is said to be making an *upper transit to the south* (see upper and lower transit, page 65). An *upper transit to the north*, means that the sun or a star crosses the meridian between the zenith and the north pole.

Example 2. The sun's altitude is $75^\circ 38'$ at upper transit to the south. Its declination is $25^\circ 33' \text{ S.}$ Find the latitude. (See Figure 55.)

When the latitude and declination are laid off on Figure 55, the zenith is south of the equator, and hence the latitude is south. For this figure:

$$h + d - L = 90^\circ$$

$$L = h + d - 90^\circ$$

$$L = 11^\circ 11' \text{ S.}$$

Formulas may be derived for the various cases of north and south transits, and north and south declinations. In general, it is better to construct a diagram like Figure 54 or Figure 55 and not to rely on formulas. Latitude may be determined in a similar manner from the meridian altitudes of the stars at upper or lower transit. Figure 56 represents a star of north declination making an upper transit to the north. In this figure,

$$h + d - L = 90^\circ$$

Problems

Find the latitude from the following meridian altitudes of the sun.

<i>1943 Date G. C. T.</i>	<i>Upper Transit</i>	<i>Declination</i>	<i>Altitude</i>
1. July 18, 20 ^h 00 ^m	North	21° 05' N.	58° 42'
2. Oct. 9, 4 ^h 00 ^m	South	5° 55' S.	42° 48'
3. Dec. 21	South	23° 27' S.	71° 27'
4. Nov. 8, 8 ^h 00 ^m	North	16° 21' S.	48° 56'
5. Aug. 2, 2 ^h 00 ^m	South	18° 02' N.	67° 16'
6. July 27, 10 ^h 00 ^m	South	19° 23' N.	20° 18'
7. Sept. 16, 18 ^h 00 ^m	North	2° 47' N.	80° 15'

Find the latitude from the meridian altitudes of the following stars.

<i>Star</i>	<i>Upper Transit</i>	<i>Declination</i>	<i>Altitude</i>
8. Vega	North	38° 44' N.	60° 21'
9. Fomalhaut	South	29° 56' S.	20° 13'
10. Capella	North	45° 56' N.	44° 04'
11. Antares	South	26° 18' S.	18° 29'
12. Arcturus	North	19° 29' N.	76° 24'
13. Sirius	South	16° 38' S.	45° 17'

Latitude from Altitudes of Polaris. The *Nautical Almanac* and the *Air Almanac* provide a short and easy method of finding the latitude from the North Star. The altitude is observed and corrected. The approximate local hour angle is found from the G. H. A. and the approximate longitude. Table III in the *Nautical Almanac*, page 284, gives a correction to apply to the altitude to obtain the latitude. This correction may be positive or negative, and is given for all values of the local hour angle.

Example. At 10 : 00 P.M. E. W. T. on January 16, 1943, the altitude of Polaris is found to be 38° 17'. There is no index error. The approximate longitude is 76°.

E. W. T.	10 : 00 P.M.	January 16
Zone Time	22 ^h 00 ^m	January 16
Zone	+ 4	
G. C. T.	2 ^h 00 ^m	January 17

<i>Nautical Almanac</i>		
G. H. A.	0 ^h	89° 20.5 (page 280)
Corr.	2 ^h	30° 04.9 (page 214)
G. H. A.		119° 25'
Longitude		76° (approximate)
L. H. A.		43° 25' (approximate)
Observed Altitude		38° 17'
Table A		— 1
Corr. Alt.		38° 16'
Table III (page 284)		— 43'
Latitude		37° 33'

The correction — 43', obtained from Table III, is found from the local hour angle.

Notice that the longitude does not need to be exact, since the corrections in Table II change very slowly, as the local hour angle changes. Hence the navigator may find his correct latitude, from his dead reckoning longitude.

The *Air Almanac* provides a similar method. In using the *Air Almanac*, find the L. H. A. φ (*i.e.*, the local hour angle of the first point of Aries) and then use the correction table on the back of the star chart.

The Circle of Position. One observation of the altitude of a celestial body does not fix position. This is true for the examples of the two previous sections. When a navigator has found his latitude, he has merely located his plane or ship on a circle on the earth. Any single observation will always locate the navigator on a circle on the earth, although not always on a parallel of latitude.

Suppose a navigator finds that the altitude of the star Vega is 66° 00' at 2^h 00^m G. C. T., May 14, 1943. From the *Nautical Almanac* he finds that Vega has a declination of 38° 44' and a G. H. A. of 342° 11'. Using the altitude, the G. H. A., and the declination, it would be easy for him to construct a *circle of position* on a globe. Vega is the zenith of a certain point on the earth; this point is called the *substellar* point of Vega. The latitude of the substellar point is 38° 44' N., the declination of Vega. The longitude of the substellar point is 342° 11' west from Greenwich or 17° 49' E. (360° — 342° 11'). The navigator's distance from the substellar point is the arc SZ of the astronomic triangle, or 90° — 66° 00' = 24° 00' (see the Astronomic Triangle, page 71). The position of point Z in the astronomic triangle is not known, and hence the navigator's position is somewhere on a small circle which has (38° 44' N., 17° 49' E.) for a pole

and a polar distance of $24^{\circ} 00'$. This small circle is shown in Figure 57. V is the substellar point of Vega; PP' the meridian of $17^{\circ} 49' E.$, and EE' the equator.

Observations of two stars at the same time would determine two circles of position. The position of the plane or ship would be one of the intersections of these two circles (see problem 3, page 7).

Suppose that navigators in different ships find the altitude of Vega at the same time. The first obtains an altitude of $66^{\circ} 00'$ and is somewhere on the circle with a polar distance of $90^{\circ} - 66^{\circ} 00'$, shown in Figure 57. The second finds an altitude of $60^{\circ} 00'$, and his ship will be on the circle of polar distance $90^{\circ} - 60^{\circ} 00'$, in Figure 57. Notice that for all the points on the *larger* circle the altitude of Vega is $60^{\circ} 00'$, and for all the points on the *smaller* circle Vega has an altitude of $66^{\circ} 00'$. *The places at which the altitude is greater are nearer the substellar point.*

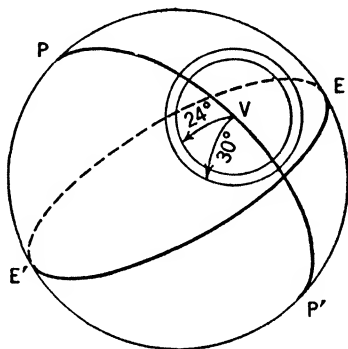


FIGURE 57.

Problems

Draw on a globe the circles of position defined as follows.

	Declination	G. H. A.	Altitude
1.	+ 30°	240°	40°
2.	+ 30°	240°	45°
3.	+ 30°	240°	35°
4.	- 20°	80°	60°
5.	- 20°	80°	65°
6.	- 20°	80°	70°

The Sumner Line. If a navigator observed the altitude of two stars at the same time, he might draw two circles of position. His ship would be at one of the points of intersection of the two circles. Figure 64, page 98, shows two position circles plotted on a Mercator map. In this case, at least, there is no doubt about which intersection is the position. It is not possible, of course, to determine with any accuracy the latitude and longitude of the intersection of two circles drawn on a globe.

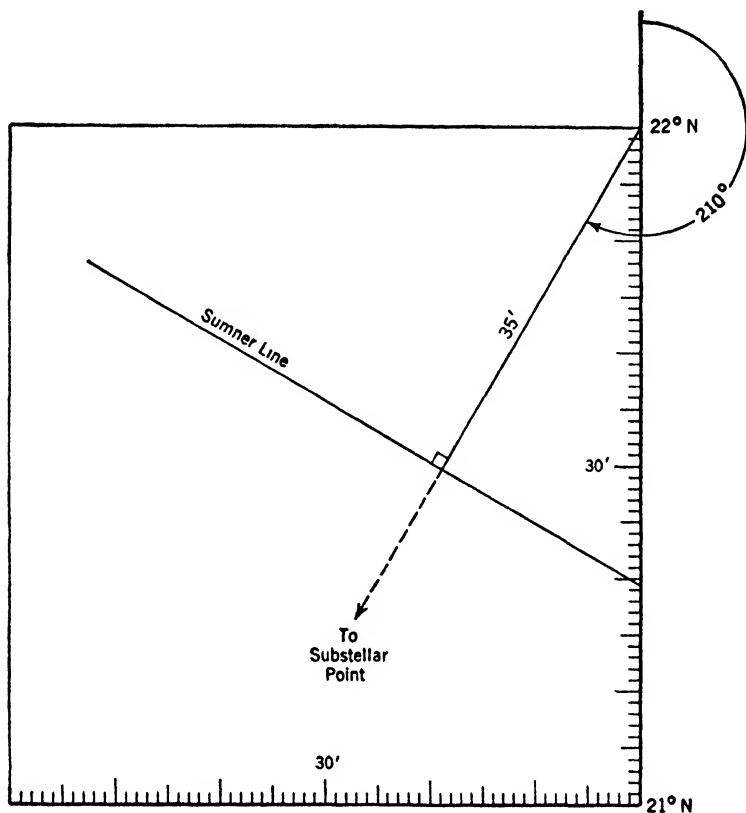


FIGURE 58.

In practice, small arcs of the position circles are drawn on a chart. Unless the observed altitude is greater than 80° , the arc of the position circle is drawn as a straight line. These *lines of position* are called *Sumner lines*, after T. H. Sumner, who discovered this method of finding position in 1837.

The Sumner line may be plotted by calculating the latitude and longitude of two points; this was the method used by Capt. Sumner. The method explained here, which is now in general use, is due to Marcq St. Hilaire, an admiral in the French Navy. The St. Hilaire method uses an assumed position; the assumed position may be that obtained by dead reckoning, or any convenient point which is near the dead-reckoning position.

In the St. Hilaire method the altitude of a star is observed. The altitude and azimuth of the star are calculated for the time of observa-

tion and for the assumed position. Sumner lines are usually plotted on Mercator charts, like the "position plotting sheets" published by the U. S. Hydrographic Office. Figure 58 shows a Sumner line plotted on a section of the position-plotting sheet for latitudes 21° to 24° .

Example 1. A navigator near (22° N., 45° W.) finds the corrected observed altitude of a star to be $45^\circ 51'$. The altitude and azimuth calculated for (22° N., 45° W.) are $45^\circ 16'$ and $210^\circ 00'$. Plot the Sumner line (see Fig. 58).

observed altitude corrected $h_o = 45^\circ 51'$

calculated altitude $h_c = 45^\circ 16'$

difference = $35'$ toward the substellar point.

Any meridian on the plotting sheet may be taken as 45° W. The line bearing 210° from (22° N., 45° W.) points to the substellar point. As one moves along the azimuth line toward the substellar point, the altitude of the star increases. The observed altitude is $35'$ greater than the calculated altitude; hence the point of observation lies on

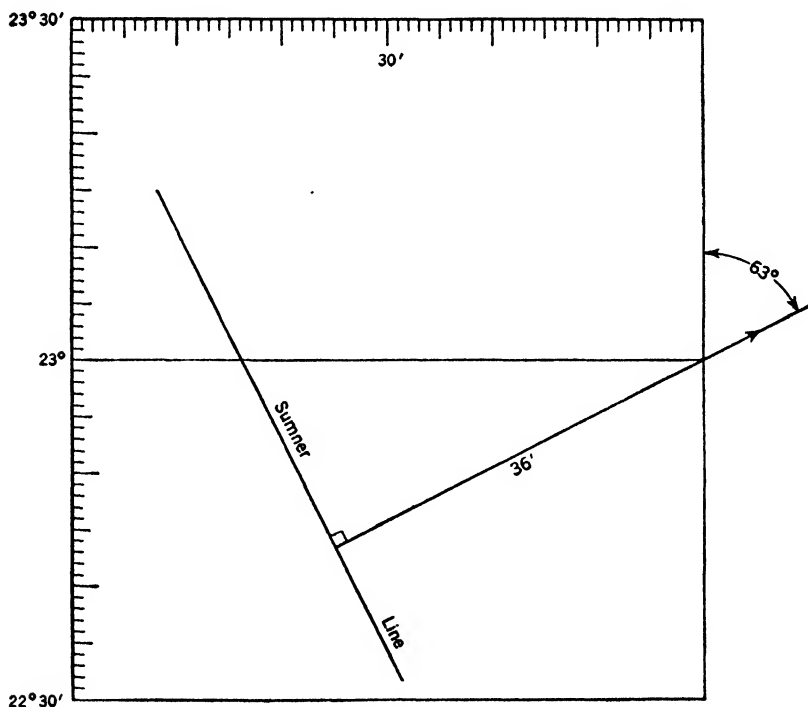


FIGURE 59.

a circle of position, which is 35' or 35 nautical miles nearer the sub-stellar point. The distance of 35 miles is laid off on the azimuth line and the Sumner line is drawn perpendicular to the azimuth line. This distance of 35' or 35 nautical miles is made equal to 35' of latitude (1 nautical mile = 1' of latitude, not longitude).

In Figure 57, the Sumner line represents a small arc of a circle of position. The azimuth line represents a small arc of the great circle joining the assumed position to the substellar point. Within the limits of the chart no appreciable error is introduced by drawing these arcs as straight lines.

Example 2. Assumed position is (23° N., 46° W.). Calculated altitude, or $h_c = 27^\circ 38'$. Calculated azimuth, or $Z_n = 63^\circ$. Corrected observed altitude, or $h_o = 27^\circ 02'$. (See Figure 59.)

$$h_o = 27^\circ 02'$$

$$h_c = 27^\circ 38'$$

$$\text{difference} = \frac{36'}{36'} \text{ away from substellar point.}$$

Since the observed altitude is less than the calculated altitude, the Sumner line is 36' or 36 nautical miles farther away from the substellar point than the assumed position.

Observed altitude *greater* than calculated altitude, Sumner line is *toward* the substellar point.

Observed altitude *less* than calculated altitude, Sumner line is *away* from substellar point.

Problems

Plot Sumner lines in the following problems. Draw two perpendicular lines for the assumed meridian and parallel. Use any convenient scale for the difference of altitude.

	Calculated Azimuth Z_n	Observed Alt. Corr. h_o	Calculated Altitude h_c
1.	135°	45° 58'	45° 48'
2.	310°	63° 27'	63° 40'
3.	70°	20° 40'	20° 30'
4.	240°	27° 08'	27° 18'
5.	195°	75° 42'	76° 00'
6.	233°	69° 47'	69° 30'
7.	150°	33° 29'	34° 00'
8.	280°	56° 18'	56° 04'

Altitude on the Prime Vertical. Plotting a Sumner line involves calculating altitude and azimuth. There is a somewhat general misconception that the sun in north latitudes never has an azimuth between north and east or between west and north. It is the purpose of this section to develop a formula for finding the latitudes at which certain stars, or the sun, at certain times of the year, have an azimuth of 90° or 270° .

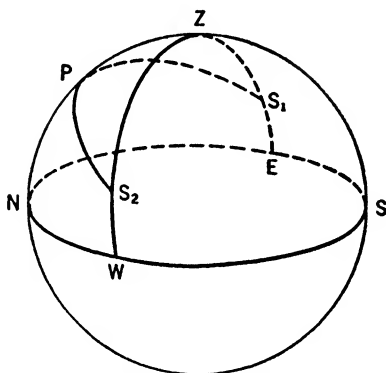


FIGURE 60.

Figure 60 shows the celestial sphere, and the line WZE passes through the zenith and the east and west points of the horizon. WZE is called the *prime vertical*. S_1 is a star which is rising and has an azimuth of 90° or due east. S_2 is a star which is setting and has an azimuth of 270° or due west. In triangle PZS_1 :

$$PZ = 90^\circ - L$$

$$PS_1 = 90^\circ - d$$

$$PZS_1 = 90^\circ$$

and $ZS_1 = 90^\circ - h$, where h is the altitude on the prime vertical.

On page 52, the *sine-cosine* formula,

$$\cos c = \cos a \cos b + \sin a \sin b \cos C,$$

was proved for spherical triangles. If in a spherical triangle $C = 90^\circ$, $\cos C = 0^\circ$, and the sine-cosine formula becomes

$$\cos c = \cos a \cos b$$

If the formula $\cos c = \cos a \cos b$ is applied to triangle PZS_1 ,

$$\cos (90^\circ - d) = \cos (90^\circ - L) \cos (90^\circ - h),$$

or

$$\sin h = \frac{\sin d}{\sin L}.$$

Example 1. At what altitude is Arcturus on the prime vertical in latitude $40^\circ 43' N$? The declination of Arcturus is $19^\circ 29' N$.

$$\sin h = \frac{\sin 19^\circ 29'}{\sin 40^\circ 43'}$$

$$\begin{aligned} \log \sin d (19^\circ 29') &= 9.52314 \\ - \log \sin L (40^\circ 43') &= 9.81446 \\ \log \sin h &= 9.70868 \\ h &= 30^\circ 45' \end{aligned}$$

Hence when Arcturus is *rising in this latitude*, for altitudes —

(a) less than $30^{\circ} 45'$, its azimuth is between 0° and 90° .

(b) greater than $30^{\circ} 45'$, its azimuth is between 90° and 180° .

When Arcturus is *setting in this latitude*, for altitudes —

(a) greater than $30^{\circ} 45'$, its azimuth is between 180° and 270° .

(b) less than $30^{\circ} 45'$, its azimuth is between 270° and 360° .

Example 2. Find the altitude at which the sun is on the prime vertical for latitude $30^{\circ} 20' N.$ on July 27, 1942. The approximate declination is $19^{\circ} 23' N.$

$$\begin{aligned}\log \sin d (19^{\circ} 23') &= 9.52099 \\ - \log \sin L (30^{\circ} 20') &= 9.70332 \\ \log \sin h &= 9.81767 \\ h &= 41^{\circ} 05'\end{aligned}$$

For this latitude and date, the sun will be east of north in the morning until it reaches an altitude of $41^{\circ} 05'$. In the afternoon it will be north of west, when its altitude is less than $41^{\circ} 05'$.

Problems

1. Find the altitude on the prime vertical for latitude $35^{\circ} N.$ for the stars, Altair ($+ 8^{\circ} 43'$), Regulus ($+ 12^{\circ} 15'$), and Aldebaran ($+ 16^{\circ} 24'$).

2. Show that stars of south declination are never observed on the prime vertical in northern latitudes. (Show that $\sin h$ is negative.)

3. Prove that the sun rises north of east and sets north of west for latitudes $23^{\circ} 27' N.$ to $66^{\circ} 33' N.$, between March 21 and September 21.

4. Restate problem 3 for south latitudes.

5. Show that a star does not cross the prime vertical in north latitudes if the star's declination is greater than the latitude.

Calculation of Altitude and Azimuth. The problem of calculating altitude is similar to the problem of finding great circle arcs. Altitude is found by the *haversine formula* and azimuth by the *sine formula*. The sides of the astronomic triangle are

$$PZ = 90^{\circ} - L, \text{ south latitudes negative}$$

$$PS = 90^{\circ} - d, \text{ south declinations negative}$$

$$ZS = 90^{\circ} - h,$$

Angle $P = t$ and is found from the G. H. A. and the longitude. The haversine formula is

$$\text{hav } c = \sin a \sin b \text{ hav } C + \text{hav } (a - b)$$

For triangle PZS, the haversine formula becomes

$$\text{hav } (90^{\circ} - h_s) = \cos L \cos d \text{ hav } t + \text{hav } (L - d).$$

Angle PZS in the astronomic triangle is either the azimuth Z_n , or $360 - Z_n$. The *sine formula* for spherical triangles is

$$\sin B = \frac{\sin C \sin b}{\sin c}$$

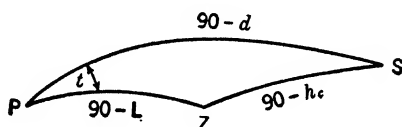


FIGURE 61.

For triangle PZS the sine formula may be written

$$\sin PZS = \frac{\sin t \cos d}{\cos h_c}$$

In using the sine formula for angle PZS, an acute angle is found from the tables; this acute angle is the acute angle from the meridian to the arc ZS. Denote this acute angle by Z ; then

$$\sin Z = \frac{\sin t \cos d}{\cos h_c}.$$

The relation between the acute angle Z and the azimuth, Z_n , is as follows:

- Star is north and east, then $Z_n = Z$.
- Star is east and south, then $Z_n = 180^\circ - Z$.
- Star is south and west, then $Z_n = 180^\circ + Z$.
- Star is west and north, then $Z_n = 360^\circ - Z$.

The relation between Z and Z_n is the same as that between angle C and the course, Figure 16, page 18.

An approximate determination of the azimuth may be made with the compass. In all but a few cases this rough observation will determine the quadrant of the azimuth. In the few doubtful cases, the altitude on the prime vertical may be computed and compared with the observed altitude (see examples 1 and 2 above); this will determine whether the star is north of east, or south of east, etc. The azimuth could be found by the haversine formula. When the haversine formula is used for finding azimuth, the angle PZS of the triangle is found and there is no ambiguity.

Example 1. Find the altitude and azimuth of the star Spica for (40° N., 50° W.) at $1^h 00^m$ G. C. T., May 3, 1943. The azimuth is between 90° and 180° .

$$\text{G. H. A. } 0^h = 19^\circ 28.1$$

$$\text{Corr. } 1^h = 15^\circ 02.5$$

$$\text{G. H. A.} = 34^\circ 31'$$

$$\text{Long.} = 50^\circ \text{ W.}$$

$$t = 15^\circ 29' \text{ east} \quad d = -10^\circ 52'$$

$$\text{hav } (90^\circ - h_c) = \cos 40^\circ \cos 10^\circ 52' \text{ hav } 15^\circ 29' + \text{hav } 50^\circ 52'$$

$$\log \cos L \quad (40^\circ 00') = 9.88425$$

$$+ \log \cos d \quad (10^\circ 52') = 9.99214$$

$$\log \text{hav } t \quad (15^\circ 29') = 8.25878$$

$$\hline 8.13517$$

$$\text{antilog } 8.13517 = .01365$$

$$+ \text{nat hav } (L-d) (50^\circ 52') = .18444$$

$$\text{nat hav } (90^\circ - h_c) = .19809$$

$$90^\circ - h_c = 52^\circ 51'$$

$$h_c = 37^\circ 09'$$

$$\sin Z = \frac{\sin 15^\circ 29' \cos 10^\circ 52'}{\cos 37^\circ 09'}$$

$$\log \sin t \quad (15^\circ 29') = 9.42644$$

$$+ \log \cos d \quad (10^\circ 52') = 9.99214$$

$$\hline 9.41858$$

$$- \log \cos h_c \quad (37^\circ 09') = 9.90149$$

$$\log \sin Z = 9.51709$$

$$Z = 19^\circ 12'$$

$$Z_n = 180^\circ - Z = 160^\circ 48'$$

In this example, $L - d = 40^\circ 00' - (-10^\circ 52') = 50^\circ 52'$. Since Spica was observed to the south and east, the acute angle Z is subtracted from 180° to obtain the azimuth.

Example 2. Find the altitude and azimuth of Regulus for the same time and position as example 1. The azimuth is between 180° and 270° .

$$\text{G. H. A. } 0^h = 68^\circ 41.0$$

$$\text{Corr. } 1^h = 15^\circ 02.5$$

$$\text{G. H. A.} = 83^\circ 44'$$

$$\text{Long.} = 50^\circ \text{ W.}$$

$$t = 33^\circ 44' \text{ west} \quad d = +12^\circ 15'$$

$$\log \text{hav } (90^\circ - h_c) = \cos 40^\circ \cos 12^\circ 15' \text{ hav } 33^\circ 44' + \text{hav } 27^\circ 45'$$

$$\log \cos L \quad (40^\circ 00') = 9.88425$$

$$+ \log \cos d \quad (12^\circ 15') = 9.99000$$

$$\log \text{hav } t \quad (33^\circ 44') = 8.92523$$

$$\hline 8.79948$$

$$\begin{aligned}
 & \text{antilog } 8.79948 = .06303 \\
 & + \text{nat hav } (L-d) (27^\circ 45') = .05751 \\
 & \text{nat hav } h_c = .12054 \\
 & 90^\circ - h_c = 40^\circ 38' \\
 & h_c = 49^\circ 22' \\
 & \sin Z = \frac{\sin 33^\circ 44' \cos 12^\circ 15'}{\cos 49^\circ 22'} \\
 & \log \sin t (33^\circ 44') = 9.74455 \\
 & + \log \cos d (12^\circ 15') = 9.99000 \\
 & \quad \quad \quad 9.73455 \\
 & - \log \cos h_c (49^\circ 22') = 9.81372 \\
 & \log \sin Z = 9.92083 \\
 & Z = 56^\circ 27' \\
 & Z_n = 180^\circ + 56^\circ 27' = 236^\circ 27'
 \end{aligned}$$

Problems

Find the altitude and azimuth in problems 1-6.

	Position	1943 Date G. C. T.	Declination	G. H. A.	Azimuth between
1. Arcturus	40° 49' N. 73° 57' W.	July 21 4 ^h 00 ^m	+ 19° 29'	144° 47'	270° & 360°
2. The Sun	22° 54' S. 43° 10' W.	Aug. 3 18 ^h 00 ^m	+ 17° 36'	88° 28'	270° & 360°
3. Fomalhaut	31° 16' N. 32° 19' E.	Aug. 15 1 ^h 00 ^m	- 29° 55'	353° 56'	180° & 270°
4. Altair	17° 30' S. 149° 30' W.	Sept. 20 8 ^h 00 ^m	+ 8° 43'	181° 20'	270° & 360°
5. Aldebaran	60° 10' N. 24° 57' E.	Nov. 24 2 ^h 00 ^m	+ 16° 24'	23° 59'	180° & 270°
6. Canopus	28° 16' S. 64° 44' W.	Dec. 1 2 ^h 00 ^m	- 52° 40'	3° 22'	90° & 180°

In problems 7-10 find first the zone and then the G. C. T. from the zone time. Find the G. H. A. and the declination from the *Nautical Almanac* or the *Air Almanac*. Calculate the altitude and azimuth for the given position.

	Position by Dead Reckoning	1943 Date Zone Time
7. Antares	22° 16' S. 166° 27' E.	Mar. 23 1 ^h 30 ^m
8. Deneb	45° 00' N. 50° 00' W.	June 4 23 ^h 00 ^m
9. Arcturus	9° 25' S. 160° 05' E.	May 18 22 ^h 00 ^m
10. Sirius	20° 10' N. 55° 06' W.	Dec. 16 2 ^h 30 ^m

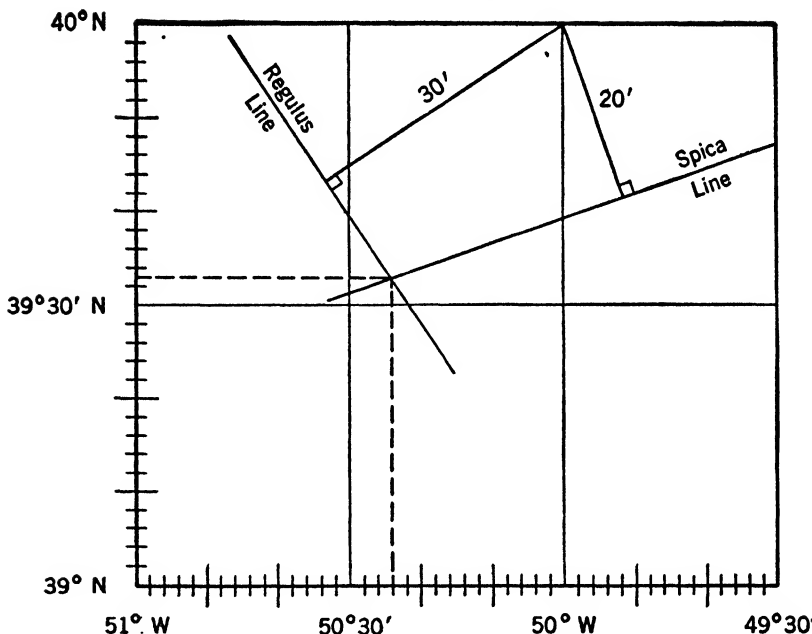


FIGURE 62.

Position by Sumner Lines. Position may be determined by the two Sumner lines resulting from two simultaneous altitudes of different stars. For the best *fix*, two stars should be selected which differ by about 90° in azimuth.

Example. A navigator in the North Atlantic finds the corrected altitude of Spica to be $37^\circ 29'$ and the corrected altitude of Regulus to be $49^\circ 52'$, at $1^{\text{h}} 00^{\text{m}}$ G. C. T., May 3, 1943. Find his position, using $(40^\circ \text{ N.}, 50^\circ \text{ W.})$ for an assumed position.

The calculated altitudes and azimuths were found in the examples of the last section.

<i>Spica</i>	<i>Regulus</i>
$h_o = 37^\circ 29'$	$h_o = 49^\circ 52'$
$h_c = 37^\circ 09'$	$h_c = 49^\circ 22'$
$20'$ toward	$30'$ toward
$Z_n = 160^\circ 48'$	$Z_n = 236^\circ 27'$

The Sumner lines from these two observations are shown on a plotting sheet in Figure 62. The ship's position determined by their intersection is $(39^\circ 33' \text{ N.}, 50^\circ 24' \text{ W.})$.

In this example the two altitudes were found at the same time. In air navigation there will generally be a time interval between the

observations. In Figure 63, the navigator of a plane has plotted line A, and then line B, but the plane has gone 45 nautical miles on a course of 225° between the times of the observations. The navigator allows for this change of position by constructing EF with a bearing of 225° and a length of 45 miles. Line C is drawn parallel to line A, and P is the position at the time of the second observation.

The two observations which were used to plot the Sumner lines of Figure 62 actually determine two circles of position. Figure 64 shows these two circles of position and the substellar points of Regulus and Spica. The circles appear distorted because they are plotted on a map. The two circles of Figure 64 have two intersections, but the intersection in the South Pacific is easily eliminated. If the difference in azimuth of two stars is near 0° or near 180° , they should not be used for a fix by Sumner lines. The best fix is obtained when the difference of azimuth is near 90° .

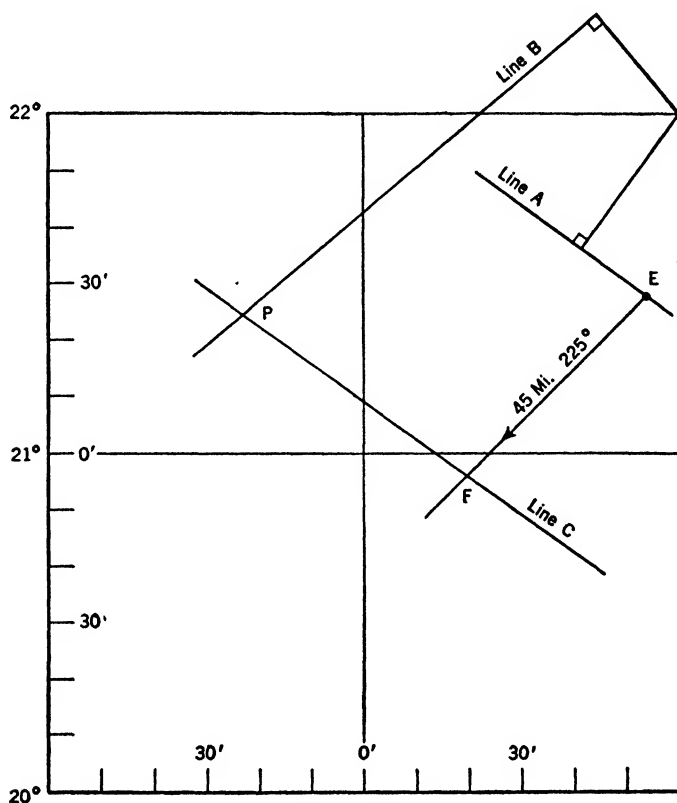


FIGURE 63.

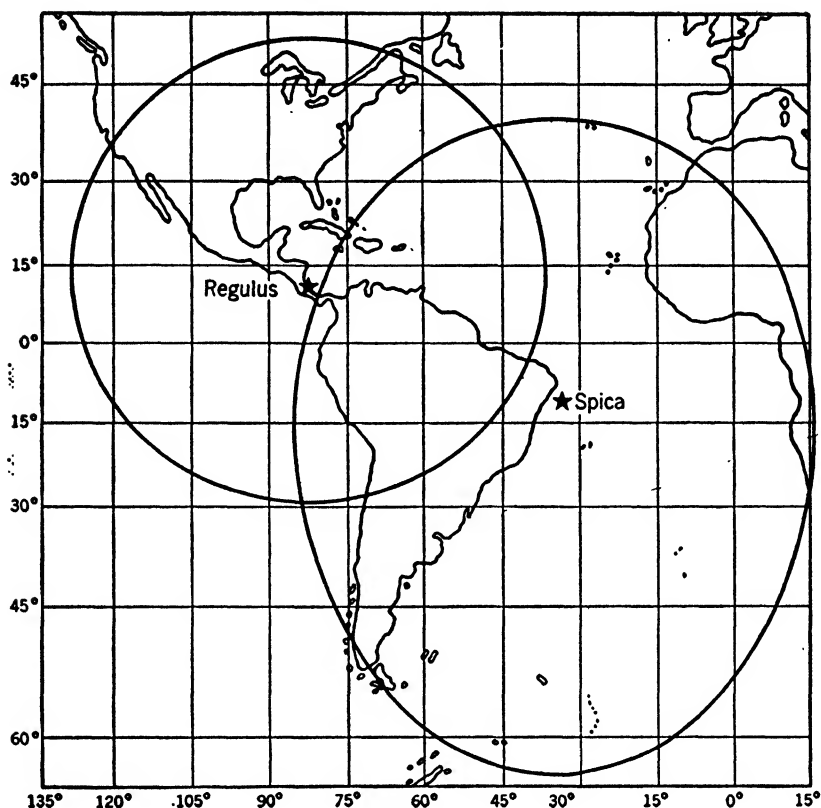


FIGURE 64.

One Sumner line is frequently valuable even though it does not fix position. If the Sumner line runs north and south, the longitude is determined. If it runs east and west, the latitude is determined. The incident which led to the discovery of the Sumner line is a striking illustration of the value of a single position line. In 1837 Captain Sumner sailed from Charleston for Greenock. As he neared his destination, his position became doubtful because he had been unable to take celestial observations. On November 27 he was able to observe a single altitude of the sun. The line of position which he plotted from this observation ran through Small's Light, and consequently the original Sumner line was the means of Sumner's ship reaching port safely.

Many recent developments have increased both the accuracy of modern navigation and the rapidity with which calculations can be

made. Numerous special tables have been prepared for calculating altitude and azimuth. The Greenwich hour angle was introduced to avoid the more cumbersome use of right ascension. The bubble sextant and the second setting watch have been developed to aid the aerial navigator. The *Air Almanac* has been published to provide all the essential astronomical data in as compact a form as possible.

Problems

Find the latitude and longitude of the point of observation in the following problems. In problems 3 and 4, find the G. H. A. and the declination from the *Nautical Almanac*. If plotting sheets are not available, they may be constructed by taking 12.00 cm. to equal 1° of longitude in each problem; then, in No. 1, 1° of latitude = 14.24 cm.; in No. 2, 1° of latitude = 13.78 cm.; in No. 3, 1° of latitude = 12.10 cm.; and in No. 4, 1° of latitude = 15.84 cm.

	<i>Assumed Position</i>	<i>1943 Date G. C. T.</i>	<i>Star</i>	<i>Dec.</i>	<i>G. H. A.</i>	<i>Obs. Alt.</i>	<i>Azimuth between</i>
1.	32° 00' N. 136° 00' W.	Aug. 10 10 ^h 00 ^m	Vega	+ 38° 44'	189° 25'	46° 44'	270° & 360°
			Fomal- haut	— 29° 55'	124° 22'	26° 47'	90° & 180°
2.	30° 00' S. 11° 00' E.	Sept. 10 21 ^h 00 ^m	Peacock	— 56° 55'	358° 43'	62° 30'	180° & 270°
			Nunki	— 26° 22'	21° 04'	61° 05'	180° & 270°
3.	10° 00' N. 60° 00' E.	Oct. 14 22 ^h 00 ^m	Observed altitude of Aldebaran = 74° 04'				
			Observed altitude of Hamal = 65° 42'				
4.	41° 00' N. 74° 00' W.	Mar. 18 00 ^h 08 ^m	Observed altitude of Regulus = 38° 58'				
			Observed altitude of Mizar = 28° 42'				

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"Historic Flights" by Bradley Jones, *Avigation*, Chapter XIV.

The Sextant:

Bowditch, 120-128; Dutton, Chapter X; *Field Mathematics* by Shuster and Bedford, Chapter XII.

Sextants and other instruments suitable for school use are sold by Lafayette Instruments, Inc., 252 Lafayette St., New York.

Plotting Sheets:

Position plotting sheets are published by the United States Hydrographic Office. The Coast and Geodetic Survey Aeronautical Charts may also be used for plotting.

Position by Sumner Lines:

Dutton, Chapters XIV, XV; Bowditch, Chapters XIV, XV; Lyon, 191-194, 203.

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No. 208, *Navigation Tables for Mariners and Aviators*, Dreisonstok.

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**FIVE PLACE
LOGARITHMS OF NUMBERS**

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AND

LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

FOR EACH MINUTE OF ARC

Pages xx-xlii

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210	32 222	32 243	32 263	32 284	32 305	32 325	32 346	32 366	32 387	32 408
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212	32 634	32 654	32 675	32 695	32 715	32 736	32 756	32 777	32 797	32 818
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214	33 041	33 062	33 082	33 102	33 122	33 143	33 163	33 183	33 203	33 224
215	33 244	33 264	33 284	33 304	33 325	33 345	33 365	33 385	33 405	33 425
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220	34 242	34 262	34 282	34 301	34 321	34 341	34 361	34 380	34 400	34 420
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222	34 635	34 655	34 674	34 694	34 713	34 733	34 753	34 772	34 792	34 811
223	34 830	34 850	34 869	34 889	34 908	34 928	34 947	34 967	34 986	35 005
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225	35 218	35 238	35 257	35 276	35 295	35 315	35 334	35 353	35 372	35 392
226	35 411	35 430	35 449	35 468	35 488	35 507	35 526	35 545	35 564	35 583
227	35 603	35 622	35 641	35 660	35 679	35 698	35 717	35 736	35 755	35 774
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229	35 984	36 003	36 021	36 040	36 059	36 078	36 097	36 116	36 135	36 154
230	36 173	36 192	36 211	36 229	36 248	36 267	36 286	36 305	36 324	36 342
231	36 361	36 380	36 399	36 418	36 436	36 455	36 474	36 493	36 511	36 530
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234	36 922	36 940	36 959	36 977	36 996	37 014	37 033	37 051	37 070	37 088
235	37 107	37 125	37 144	37 162	37 181	37 199	37 218	37 236	37 254	37 273
236	37 291	37 310	37 328	37 346	37 365	37 383	37 401	37 420	37 438	37 457
237	37 475	37 493	37 511	37 530	37 548	37 566	37 585	37 603	37 621	37 639
238	37 658	37 676	37 694	37 712	37 731	37 749	37 767	37 785	37 803	37 822
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247	39 270	39 287	39 305	39 322	39 340	39 358	39 375	39 393	39 410	39 428
248	39 445	39 463	39 480	39 498	39 515	39 533	39 550	39 568	39 585	39 602
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251	39 967	39 985	40 002	40 019	40 037	40 054	40 071	40 088	40 106	40 123
252	40 140	40 157	40 175	40 192	40 209	40 226	40 243	40 261	40 278	40 295
253	40 312	40 329	40 346	40 364	40 381	40 398	40 415	40 432	40 449	40 466
254	40 483	40 500	40 518	40 535	40 552	40 569	40 586	40 603	40 620	40 637
255	40 654	40 671	40 688	40 705	40 722	40 739	40 756	40 773	40 790	40 807
256	40 824	40 841	40 858	40 875	40 892	40 909	40 926	40 943	40 960	40 976
257	40 993	41 010	41 027	41 044	41 061	41 078	41 095	41 111	41 128	41 145
258	41 162	41 179	41 196	41 212	41 229	41 246	41 263	41 280	41 296	41 313
259	41 330	41 347	41 363	41 380	41 397	41 414	41 430	41 447	41 464	41 481
260	41 497	41 514	41 531	41 547	41 564	41 581	41 597	41 614	41 631	41 647
261	41 664	41 681	41 697	41 714	41 731	41 747	41 764	41 780	41 797	41 814
262	41 830	41 847	41 863	41 880	41 896	41 913	41 929	41 946	41 963	41 979
263	41 996	42 012	42 029	42 045	42 062	42 078	42 095	42 111	42 127	42 144
264	42 160	42 177	42 193	42 210	42 226	42 243	42 259	42 275	42 292	42 308
265	42 325	42 341	42 357	42 374	42 390	42 406	42 423	42 439	42 455	42 472
266	42 488	42 504	42 521	42 537	42 553	42 570	42 586	42 602	42 619	42 635
267	42 651	42 667	42 684	42 700	42 716	42 732	42 749	42 765	42 781	42 797
268	42 813	42 830	42 846	42 862	42 878	42 894	42 911	42 927	42 943	42 959
269	42 975	42 991	43 008	43 024	43 040	43 056	43 072	43 088	43 104	43 120
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271	43 297	43 313	43 329	43 345	43 361	43 377	43 393	43 409	43 425	43 441
272	43 457	43 473	43 489	43 505	43 521	43 537	43 553	43 569	43 584	43 600
273	43 616	43 632	43 648	43 664	43 680	43 696	43 712	43 727	43 743	43 759
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277	44 248	44 264	44 279	44 295	44 311	44 326	44 342	44 358	44 373	44 389
278	44 404	44 420	44 436	44 451	44 467	44 483	44 498	44 514	44 529	44 545
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281	44 871	44 886	44 902	44 917	44 932	44 948	44 963	44 979	44 994	45 010
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283	45 179	45 194	45 209	45 225	45 240	45 255	45 271	45 286	45 301	45 317
284	45 332	45 347	45 362	45 378	45 393	45 408	45 423	45 439	45 454	45 469
285	45 484	45 500	45 515	45 530	45 545	45 561	45 576	45 591	45 606	45 621
286	45 637	45 652	45 667	45 682	45 697	45 712	45 728	45 743	45 758	45 773
287	45 788	45 803	45 818	45 834	45 849	45 864	45 879	45 894	45 909	45 924
288	45 939	45 954	45 969	45 984	46 000	46 015	46 030	46 045	46 060	46 075
289	46 090	46 105	46 120	46 135	46 150	46 165	46 180	46 195	46 210	46 225
290	46 240	46 255	46 270	46 285	46 300	46 315	46 330	46 345	46 359	46 374
291	46 389	46 404	46 419	46 434	46 449	46 464	46 479	46 494	46 509	46 523
292	46 538	46 553	46 568	46 583	46 598	46 613	46 627	46 642	46 657	46 672
293	46 687	46 702	46 716	46 731	46 746	46 761	46 776	46 790	46 805	46 820
294	46 835	46 850	46 864	46 879	46 894	46 909	46 923	46 938	46 953	46 967
295	46 982	46 997	47 012	47 026	47 041	47 056	47 070	47 085	47 100	47 114
296	47 129	47 144	47 159	47 173	47 188	47 202	47 217	47 232	47 246	47 261
297	47 276	47 290	47 305	47 319	47 334	47 349	47 363	47 378	47 392	47 407
298	47 422	47 436	47 451	47 465	47 480	47 494	47 509	47 524	47 538	47 553
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301	47 857	47 871	47 885	47 900	47 914	47 929	47 943	47 958	47 972	47 986
302	48 001	48 015	48 029	48 044	48 058	48 073	48 087	48 101	48 116	48 130
303	48 144	48 159	48 173	48 187	48 202	48 216	48 230	48 244	48 259	48 273
304	48 287	48 302	48 316	48 330	48 344	48 359	48 373	48 387	48 401	48 416
305	48 430	48 444	48 458	48 473	48 487	48 501	48 515	48 530	48 544	48 558
306	48 572	48 586	48 601	48 615	48 629	48 643	48 657	48 671	48 686	48 700
307	48 714	48 728	48 742	48 756	48 770	48 785	48 799	48 813	48 827	48 841
308	48 855	48 869	48 883	48 897	48 911	48 926	48 940	48 954	48 968	48 982
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310	49 136	49 150	49 164	49 178	49 192	49 206	49 220	49 234	49 248	49 262
311	49 276	49 290	49 304	49 318	49 332	49 346	49 360	49 374	49 388	49 402
312	49 415	49 429	49 443	49 457	49 471	49 485	49 499	49 513	49 527	49 541
313	49 554	49 568	49 582	49 596	49 610	49 624	49 638	49 651	49 665	49 679
314	49 693	49 707	49 721	49 734	49 748	49 762	49 776	49 790	49 803	49 817
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316	49 969	49 982	49 996	50 010	50 024	50 037	50 051	50 065	50 079	50 092
317	50 106	50 120	50 133	50 147	50 161	50 174	50 188	50 202	50 215	50 229
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321	50 651	50 664	50 678	50 691	50 705	50 718	50 732	50 745	50 759	50 772
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324	51 055	51 068	51 081	51 095	51 108	51 121	51 135	51 148	51 162	51 175
325	51 188	51 202	51 215	51 228	51 242	51 255	51 268	51 282	51 295	51 308
326	51 322	51 335	51 348	51 362	51 375	51 388	51 402	51 415	51 428	51 441
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329	51 720	51 733	51 746	51 759	51 772	51 786	51 799	51 812	51 825	51 838
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337	52 763	52 776	52 789	52 802	52 815	52 827	52 840	52 853	52 866	52 879
338	52 892	52 905	52 917	52 930	52 943	52 956	52 969	52 982	52 994	53 007
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340	53 148	53 161	53 173	53 186	53 199	53 212	53 224	53 237	53 250	53 263
341	53 275	53 288	53 301	53 314	53 326	53 339	53 352	53 364	53 377	53 390
342	53 403	53 415	53 428	53 441	53 453	53 466	53 479	53 491	53 504	53 517
343	53 529	53 542	53 555	53 567	53 580	53 593	53 605	53 618	53 631	53 643
344	53 656	53 668	53 681	53 694	53 706	53 719	53 732	53 744	53 757	53 769
345	53 782	53 794	53 807	53 820	53 832	53 845	53 857	53 870	53 882	53 895
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348	54 158	54 170	54 183	54 195	54 208	54 220	54 233	54 245	54 258	54 270
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353	54 777	54 790	54 802	54 814	54 827	54 839	54 851	54 864	54 876	54 888
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356	55 145	55 157	55 169	55 182	55 194	55 206	55 218	55 230	55 242	55 255
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360	55 630	55 642	55 654	55 666	55 678	55 691	55 703	55 715	55 727	55 739
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365	56 229	56 241	56 253	56 265	56 277	56 289	56 301	56 312	56 324	56 336
366	56 348	56 360	56 372	56 384	56 396	56 407	56 419	56 431	56 443	56 455
367	56 467	56 478	56 490	56 502	56 514	56 526	56 538	56 549	56 561	56 573
368	56 585	56 597	56 608	56 620	56 632	56 644	56 656	56 667	56 679	56 691
369	56 703	56 714	56 726	56 738	56 750	56 761	56 773	56 785	56 797	56 808
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372	57 054	57 066	57 078	57 089	57 101	57 113	57 124	57 136	57 148	57 159
373	57 171	57 183	57 194	57 206	57 217	57 229	57 241	57 252	57 264	57 276
374	57 287	57 299	57 310	57 322	57 334	57 345	57 357	57 368	57 380	57 392
375	57 403	57 415	57 426	57 438	57 449	57 461	57 473	57 484	57 496	57 507
376	57 519	57 530	57 542	57 553	57 565	57 576	57 588	57 600	57 611	57 623
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380	57 978	57 990	58 001	58 013	58 024	58 035	58 047	58 058	58 070	58 081
381	58 092	58 104	58 115	58 127	58 138	58 149	58 161	58 172	58 184	58 195
382	58 206	58 218	58 229	58 240	58 252	58 263	58 274	58 286	58 297	58 309
383	58 320	58 331	58 343	58 354	58 365	58 377	58 388	58 399	58 410	58 422
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385	58 546	58 557	58 569	58 580	58 591	58 602	58 614	58 625	58 636	58 647
386	58 659	58 670	58 681	58 692	58 704	58 715	58 726	58 737	58 749	58 760
387	58 771	58 782	58 794	58 805	58 816	58 827	58 838	58 850	58 861	58 872
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390	59 106	59 118	59 129	59 140	59 151	59 162	59 173	59 184	59 195	59 207
391	59 218	59 229	59 240	59 251	59 262	59 273	59 284	59 295	59 306	59 318
392	59 329	59 340	59 351	59 362	59 373	59 384	59 395	59 406	59 417	59 428
393	59 439	59 450	59 461	59 472	59 483	59 494	59 506	59 517	59 528	59 539
394	59 550	59 561	59 572	59 583	59 594	59 605	59 616	59 627	59 638	59 649
395	59 660	59 671	59 682	59 693	59 704	59 715	59 726	59 737	59 748	59 759
396	59 770	59 780	59 791	59 802	59 813	59 824	59 835	59 846	59 857	59 868
397	59 879	59 890	59 901	59 912	59 923	59 934	59 945	59 956	59 966	59 977
398	59 988	59 999	60 010	60 021	60 032	60 043	60 054	60 065	60 076	60 086
399	60 097	60 108	60 119	60 130	60 141	60 152	60 163	60 173	60 184	60 195
No.	0	1	2	3	4	5	6	7	8	9

No.	0	1	2	3	4	5	6	7	8	9
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401	60 314	60 325	60 336	60 347	60 358	60 369	60 379	60 390	60 401	60 412
402	60 423	60 433	60 444	60 455	60 466	60 477	60 487	60 498	60 509	60 520
403	60 531	60 541	60 552	60 563	60 574	60 584	60 595	60 606	60 617	60 627
404	60 638	60 649	60 660	60 670	60 681	60 692	60 703	60 713	60 724	60 735
405	60 746	60 756	60 767	60 778	60 788	60 799	60 810	60 821	60 831	60 842
406	60 853	60 863	60 874	60 885	60 895	60 906	60 917	60 927	60 938	60 949
407	60 959	60 970	60 981	60 991	61 002	61 013	61 023	61 034	61 045	61 055
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409	61 172	61 183	61 194	61 204	61 215	61 225	61 236	61 247	61 257	61 268
410	61 278	61 289	61 300	61 310	61 321	61 331	61 342	61 352	61 363	61 374
411	61 384	61 395	61 405	61 416	61 426	61 437	61 448	61 458	61 469	61 479
412	61 490	61 500	61 511	61 521	61 532	61 542	61 553	61 563	61 574	61 584
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414	61 700	61 711	61 721	61 731	61 742	61 752	61 763	61 773	61 784	61 794
415	61 805	61 815	61 826	61 836	61 847	61 857	61 868	61 878	61 888	61 899
416	61 909	61 920	61 930	61 941	61 951	61 962	61 972	61 982	61 993	62 003
417	62 014	62 024	62 034	62 045	62 055	62 066	62 076	62 086	62 097	62 107
418	62 118	62 128	62 138	62 149	62 159	62 170	62 180	62 190	62 201	62 211
419	62 221	62 232	62 242	62 252	62 263	62 273	62 284	62 294	62 304	62 315
420	62 325	62 335	62 346	62 356	62 366	62 377	62 387	62 397	62 408	62 418
421	62 428	62 439	62 449	62 459	62 469	62 480	62 490	62 500	62 511	62 521
422	62 531	62 542	62 552	62 562	62 572	62 583	62 593	62 603	62 613	62 624
423	62 634	62 644	62 655	62 665	62 675	62 685	62 696	62 706	62 716	62 726
424	62 737	62 747	62 757	62 767	62 778	62 788	62 798	62 808	62 818	62 829
425	62 839	62 849	62 859	62 870	62 880	62 890	62 900	62 910	62 921	62 931
426	62 941	62 951	62 961	62 972	62 982	62 992	63 002	63 012	63 022	63 033
427	63 043	63 053	63 063	63 073	63 083	63 094	63 104	63 114	63 124	63 134
428	63 144	63 155	63 165	63 175	63 185	63 195	63 205	63 215	63 225	63 236
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432	63 548	63 558	63 568	63 579	63 589	63 599	63 609	63 619	63 629	63 639
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438	64 147	64 157	64 167	64 177	64 187	64 197	64 207	64 217	64 227	64 237
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441	64 444	64 454	64 464	64 473	64 483	64 493	64 503	64 513	64 523	64 532
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454	65 706	65 715	65 725	65 734	65 744	65 753	65 763	65 772	65 782	65 792
455	65 801	65 811	65 820	65 830	65 839	65 849	65 858	65 868	65 877	65 887
456	65 896	65 906	65 916	65 925	65 935	65 944	65 954	65 963	65 973	65 982
457	65 992	66 001	66 011	66 020	66 030	66 039	66 049	66 058	66 068	66 077
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493	69 285	69 294	69 302	69 311	69 320	69 329	69 338	69 346	69 355	69 364
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495	69 461	69 469	69 478	69 487	69 496	69 504	69 513	69 522	69 531	69 539
496	69 548	69 557	69 566	69 574	69 583	69 592	69 601	69 609	69 618	69 627
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503	70 157	70 165	70 174	70 183	70 191	70 200	70 209	70 217	70 226	70 234
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552	74 194	74 202	74 210	74 218	74 225	74 233	74 241	74 249	74 257	74 265
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571	75 664	75 671	75 679	75 686	75 694	75 702	75 709	75 717	75 724	75 732
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573	75 815	75 823	75 831	75 838	75 846	75 853	75 861	75 868	75 876	75 884
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575	75 967	75 974	75 982	75 989	75 997	76 005	76 012	76 020	76 027	76 035
576	76 042	76 050	76 057	76 065	76 072	76 080	76 087	76 095	76 103	76 110
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587	76 864	76 871	76 879	76 886	76 893	76 901	76 908	76 916	76 923	76 930
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616	78 958	78 965	78 972	78 979	78 986	78 993	79 000	79 007	79 014	79 021
617	79 029	79 036	79 043	79 050	79 057	79 064	79 071	79 078	79 085	79 092
618	79 099	79 106	79 113	79 120	79 127	79 134	79 141	79 148	79 155	79 162
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622	79 379	79 386	79 393	79 400	79 407	79 414	79 421	79 428	79 435	79 442
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626	79 657	79 664	79 671	79 678	79 685	79 692	79 699	79 706	79 713	79 720
627	79 727	79 734	79 741	79 748	79 754	79 761	79 768	79 775	79 782	79 789
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633	80 140	80 147	80 154	80 161	80 168	80 175	80 182	80 188	80 195	80 202
634	80 209	80 216	80 223	80 229	80 236	80 243	80 250	80 257	80 264	80 271
635	80 277	80 284	80 291	80 298	80 305	80 312	80 318	80 325	80 332	80 339
636	80 346	80 353	80 359	80 366	80 373	80 380	80 387	80 393	80 400	80 407
637	80 414	80 421	80 428	80 434	80 441	80 448	80 455	80 462	80 468	80 475
638	80 482	80 489	80 496	80 502	80 509	80 516	80 523	80 530	80 536	80 543
639	80 550	80 557	80 564	80 570	80 577	80 584	80 591	80 598	80 604	80 611
640	80 618	80 625	80 632	80 638	80 645	80 652	80 659	80 665	80 672	80 679
641	80 686	80 693	80 699	80 706	80 713	80 720	80 726	80 733	80 740	80 747
642	80 754	80 760	80 767	80 774	80 781	80 787	80 794	80 801	80 808	80 814
643	80 821	80 828	80 835	80 841	80 848	80 855	80 862	80 868	80 875	80 882
644	80 889	80 895	80 902	80 909	80 916	80 922	80 929	80 936	80 943	80 949
645	80 956	80 963	80 969	80 976	80 983	80 990	80 996	81 003	81 010	81 017
646	81 023	81 030	81 037	81 043	81 050	81 057	81 064	81 070	81 077	81 084
647	81 090	81 097	81 104	81 111	81 117	81 124	81 131	81 137	81 144	81 151
648	81 158	81 164	81 171	81 178	81 184	81 191	81 198	81 204	81 211	81 218
649	81 224	81 231	81 238	81 245	81 251	81 258	81 265	81 271	81 278	81 285
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No.	0	1	2	3	4	5	6	7	8	9
650	81 291	81 298	81 305	81 311	81 318	81 325	81 331	81 338	81 345	81 351
651	81 358	81 365	81 371	81 378	81 385	81 391	81 398	81 405	81 411	81 418
652	81 425	81 431	81 438	81 445	81 451	81 458	81 465	81 471	81 478	81 485
653	81 491	81 498	81 505	81 511	81 518	81 525	81 531	81 538	81 544	81 551
654	81 558	81 564	81 571	81 578	81 584	81 591	81 598	81 604	81 611	81 617
655	81 624	81 631	81 637	81 644	81 651	81 657	81 664	81 671	81 677	81 684
656	81 690	81 697	81 704	81 710	81 717	81 723	81 730	81 737	81 743	81 750
657	81 757	81 763	81 770	81 776	81 783	81 790	81 796	81 803	81 809	81 816
658	81 823	81 829	81 836	81 842	81 849	81 856	81 862	81 869	81 875	81 882
659	81 889	81 895	81 902	81 908	81 915	81 921	81 928	81 935	81 941	81 948
660	81 954	81 961	81 968	81 974	81 981	81 987	81 994	82 000	82 007	82 014
661	82 020	82 027	82 033	82 040	82 046	82 053	82 060	82 066	82 073	82 079
662	82 086	82 092	82 099	82 105	82 112	82 119	82 125	82 132	82 138	82 145
663	82 151	82 158	82 164	82 171	82 178	82 184	82 191	82 197	82 204	82 210
664	82 217	82 223	82 230	82 236	82 243	82 249	82 256	82 263	82 269	82 276
665	82 282	82 289	82 295	82 302	82 308	82 315	82 321	82 328	82 334	82 341
666	82 347	82 354	82 360	82 367	82 373	82 380	82 387	82 393	82 400	82 406
667	82 413	82 419	82 426	82 432	82 439	82 445	82 452	82 458	82 465	82 471
668	82 478	82 484	82 491	82 497	82 504	82 510	82 517	82 523	82 530	82 536
669	82 543	82 549	82 556	82 562	82 569	82 575	82 582	82 588	82 595	82 601
670	82 607	82 614	82 620	82 627	82 633	82 640	82 646	82 653	82 659	82 666
671	82 672	82 679	82 685	82 692	82 698	82 705	82 711	82 718	82 724	82 730
672	82 737	82 743	82 750	82 756	82 763	82 769	82 776	82 782	82 789	82 795
673	82 802	82 808	82 814	82 821	82 827	82 834	82 840	82 847	82 853	82 860
674	82 866	82 872	82 879	82 885	82 892	82 898	82 905	82 911	82 918	82 924
675	82 930	82 937	82 943	82 950	82 956	82 963	82 969	82 975	82 982	82 988
676	82 995	83 001	83 008	83 014	83 020	83 027	83 033	83 040	83 046	83 052
677	83 059	83 065	83 072	83 078	83 085	83 091	83 097	83 104	83 110	83 117
678	83 123	83 129	83 136	83 142	83 149	83 155	83 161	83 168	83 174	83 181
679	83 187	83 193	83 200	83 206	83 213	83 219	83 225	83 232	83 238	83 245
680	83 251	83 257	83 264	83 270	83 276	83 283	83 289	83 296	83 302	83 308
681	83 315	83 321	83 327	83 334	83 340	83 347	83 353	83 359	83 366	83 372
682	83 378	83 385	83 391	83 398	83 404	83 410	83 417	83 423	83 429	83 436
683	83 442	83 448	83 455	83 461	83 467	83 474	83 480	83 487	83 493	83 499
684	83 506	83 512	83 518	83 525	83 531	83 537	83 544	83 550	83 556	83 563
685	83 569	83 575	83 582	83 588	83 594	83 601	83 607	83 613	83 620	83 626
686	83 632	83 639	83 645	83 651	83 658	83 664	83 670	83 677	83 683	83 689
687	83 696	83 702	83 708	83 715	83 721	83 727	83 734	83 740	83 746	83 753
688	83 759	83 765	83 771	83 778	83 784	83 790	83 797	83 803	83 809	83 816
689	83 822	83 828	83 835	83 841	83 847	83 853	83 860	83 866	83 872	83 879
690	83 885	83 891	83 897	83 904	83 910	83 916	83 923	83 929	83 935	83 942
691	83 948	83 954	83 960	83 967	83 973	83 979	83 985	83 992	83 998	84 004
692	84 011	84 017	84 023	84 029	84 036	84 042	84 048	84 055	84 061	84 067
693	84 073	84 080	84 086	84 092	84 098	84 105	84 111	84 117	84 123	84 130
694	84 136	84 142	84 148	84 155	84 161	84 167	84 173	84 180	84 186	84 192
695	84 198	84 205	84 211	84 217	84 223	84 230	84 236	84 242	84 248	84 255
696	84 261	84 267	84 273	84 280	84 286	84 292	84 298	84 305	84 311	84 317
697	84 323	84 330	84 336	84 342	84 348	84 354	84 361	84 367	84 373	84 379
698	84 386	84 392	84 398	84 404	84 410	84 417	84 423	84 429	84 435	84 442
699	84 448	84 454	84 460	84 466	84 473	84 479	84 485	84 491	84 497	84 504
No.	0	1	2	3	4	5	6	7	8	9

No.	0	1	2	3	4	5	6	7	8	9
700	84 510	84 516	84 522	84 528	84 535	84 541	84 547	84 553	84 559	84 566
701	84 572	84 578	84 584	84 590	84 597	84 603	84 609	84 615	84 621	84 628
702	84 634	84 640	84 646	84 652	84 658	84 665	84 671	84 677	84 683	84 689
703	84 696	84 702	84 708	84 714	84 720	84 726	84 733	84 739	84 745	84 751
704	84 757	84 763	84 770	84 776	84 782	84 788	84 794	84 800	84 807	84 813
705	84 819	84 825	84 831	84 837	84 844	84 850	84 856	84 862	84 868	84 874
706	84 880	84 887	84 893	84 899	84 905	84 911	84 917	84 924	84 930	84 936
707	84 942	84 948	84 954	84 960	84 967	84 973	84 979	84 985	84 991	84 997
708	85 003	85 009	85 016	85 022	85 028	85 034	85 040	85 046	85 052	85 058
709	85 065	85 071	85 077	85 083	85 089	85 095	85 101	85 107	85 114	85 120
710	85 126	85 132	85 138	85 144	85 150	85 156	85 163	85 169	85 175	85 181
711	85 187	85 193	85 199	85 205	85 211	85 217	85 224	85 230	85 236	85 242
712	85 248	85 254	85 260	85 266	85 272	85 278	85 285	85 291	85 297	85 303
713	85 309	85 315	85 321	85 327	85 333	85 339	85 345	85 352	85 358	85 364
714	85 370	85 376	85 382	85 388	85 394	85 400	85 406	85 412	85 418	85 425
715	85 431	85 437	85 443	85 449	85 455	85 461	85 467	85 473	85 479	85 485
716	85 491	85 497	85 503	85 509	85 516	85 522	85 528	85 534	85 540	85 546
717	85 552	85 558	85 564	85 570	85 576	85 582	85 588	85 594	85 600	85 606
718	85 612	85 618	85 625	85 631	85 637	85 643	85 649	85 655	85 661	85 667
719	85 673	85 679	85 685	85 691	85 697	85 703	85 709	85 715	85 721	85 727
720	85 733	85 739	85 745	85 751	85 757	85 763	85 769	85 775	85 781	85 788
721	85 794	85 800	85 806	85 812	85 818	85 824	85 830	85 836	85 842	85 848
722	85 854	85 860	85 866	85 872	85 878	85 884	85 890	85 896	85 902	85 908
723	85 914	85 920	85 926	85 932	85 938	85 944	85 950	85 956	85 962	85 968
724	85 974	85 980	85 986	85 992	85 998	86 004	86 010	86 016	86 022	86 028
725	86 034	86 040	86 046	86 052	86 058	86 064	86 070	86 076	86 082	86 088
726	86 094	86 100	86 106	86 112	86 118	86 124	86 130	86 136	86 141	86 147
727	86 153	86 159	86 165	86 171	86 177	86 183	86 189	86 195	86 201	86 207
728	86 213	86 219	86 225	86 231	86 237	86 243	86 249	86 255	86 261	86 267
729	86 273	86 279	86 285	86 291	86 297	86 303	86 308	86 314	86 320	86 326
730	86 332	86 338	86 344	86 350	86 356	86 362	86 368	86 374	86 380	86 386
731	86 392	86 398	86 404	86 410	86 415	86 421	86 427	86 433	86 439	86 445
732	86 451	86 457	86 463	86 469	86 475	86 481	86 487	86 493	86 499	86 504
733	86 510	86 516	86 522	86 528	86 534	86 540	86 546	86 552	86 558	86 564
734	86 570	86 576	86 581	86 587	86 593	86 599	86 605	86 611	86 617	86 623
735	86 629	86 635	86 641	86 646	86 652	86 658	86 664	86 670	86 676	86 682
736	86 688	86 694	86 700	86 705	86 711	86 717	86 723	86 729	86 735	86 741
737	86 747	86 753	86 759	86 764	86 770	86 776	86 782	86 788	86 794	86 800
738	86 806	86 812	86 817	86 823	86 829	86 835	86 841	86 847	86 853	86 859
739	86 864	86 870	86 876	86 882	86 888	86 894	86 900	86 906	86 911	86 917
740	86 923	86 929	86 935	86 941	86 947	86 953	86 958	86 964	86 970	86 976
741	86 982	86 988	86 994	86 999	87 005	87 011	87 017	87 023	87 029	87 035
742	87 040	87 046	87 052	87 058	87 064	87 070	87 075	87 081	87 087	87 093
743	87 099	87 105	87 111	87 116	87 122	87 128	87 134	87 140	87 146	87 151
744	87 157	87 163	87 169	87 175	87 181	87 186	87 192	87 198	87 204	87 210
745	87 216	87 221	87 227	87 233	87 239	87 245	87 251	87 256	87 262	87 268
746	87 274	87 280	87 286	87 291	87 297	87 303	87 309	87 315	87 320	87 326
747	87 332	87 338	87 344	87 349	87 355	87 361	87 367	87 373	87 379	87 384
748	87 390	87 396	87 402	87 408	87 413	87 419	87 425	87 431	87 437	87 442
749	87 448	87 454	87 460	87 466	87 471	87 477	87 483	87 489	87 495	87 500
No.	0	1	2	3	4	5	6	7	8	9

No.	0	1	2	3	4	5	6	7	8	9
750	87 506	87 512	87 518	87 523	87 529	87 535	87 541	87 547	87 552	87 558
751	87 564	87 570	87 576	87 581	87 587	87 593	87 599	87 604	87 610	87 616
752	87 622	87 628	87 633	87 639	87 645	87 651	87 656	87 662	87 668	87 674
753	87 679	87 685	87 691	87 697	87 703	87 708	87 714	87 720	87 726	87 731
754	87 737	87 743	87 749	87 754	87 760	87 766	87 772	87 777	87 783	87 789
755	87 795	87 800	87 806	87 812	87 818	87 823	87 829	87 835	87 841	87 846
756	87 852	87 858	87 864	87 869	87 875	87 881	87 887	87 892	87 898	87 904
757	87 910	87 915	87 921	87 927	87 933	87 938	87 944	87 950	87 955	87 961
758	87 967	87 973	87 978	87 984	87 990	87 996	88 001	88 007	88 013	88 018
759	88 024	88 030	88 036	88 041	88 047	88 053	88 058	88 064	88 070	88 076
760	88 081	88 087	88 093	88 098	88 104	88 110	88 116	88 121	88 127	88 133
761	88 138	88 144	88 150	88 156	88 161	88 167	88 173	88 178	88 184	88 190
762	88 195	88 201	88 207	88 213	88 218	88 224	88 230	88 235	88 241	88 247
763	88 252	88 258	88 264	88 270	88 275	88 281	88 287	88 292	88 298	88 304
764	88 309	88 315	88 321	88 326	88 332	88 338	88 343	88 349	88 355	88 360
765	88 366	88 372	88 377	88 383	88 389	88 395	88 400	88 406	88 412	88 417
766	88 423	88 429	88 434	88 440	88 446	88 451	88 457	88 463	88 468	88 474
767	88 480	88 485	88 491	88 497	88 502	88 508	88 513	88 519	88 525	88 530
768	88 536	88 542	88 547	88 553	88 559	88 564	88 570	88 576	88 581	88 587
769	88 593	88 598	88 604	88 610	88 615	88 621	88 627	88 632	88 638	88 643
770	88 649	88 655	88 660	88 666	88 672	88 677	88 683	88 689	88 694	88 700
771	88 705	88 711	88 717	88 722	88 728	88 734	88 739	88 745	88 750	88 756
772	88 762	88 767	88 773	88 779	88 784	88 790	88 795	88 801	88 807	88 812
773	88 818	88 824	88 829	88 835	88 840	88 846	88 852	88 857	88 863	88 868
774	88 874	88 880	88 885	88 891	88 897	88 902	88 908	88 913	88 919	88 925
775	88 930	88 936	88 941	88 947	88 953	88 958	88 964	88 969	88 975	88 981
776	88 986	88 992	88 997	89 003	89 009	89 014	89 020	89 025	89 031	89 037
777	89 042	89 048	89 053	89 059	89 064	89 070	89 076	89 081	89 087	89 092
778	89 098	89 104	89 109	89 115	89 120	89 126	89 131	89 137	89 143	89 148
779	89 154	89 159	89 165	89 170	89 176	89 182	89 187	89 193	89 198	89 204
780	89 209	89 215	89 221	89 226	89 232	89 237	89 243	89 248	89 254	89 260
781	89 265	89 271	89 276	89 282	89 287	89 293	89 298	89 304	89 310	89 315
782	89 321	89 326	89 332	89 337	89 343	89 348	89 354	89 360	89 365	89 371
783	89 376	89 382	89 387	89 393	89 398	89 404	89 409	89 415	89 421	89 426
784	89 432	89 437	89 443	89 448	89 454	89 459	89 465	89 470	89 476	89 481
785	89 487	89 492	89 498	89 504	89 509	89 515	89 520	89 526	89 531	89 537
786	89 542	89 548	89 553	89 559	89 564	89 570	89 575	89 581	89 586	89 592
787	89 597	89 603	89 609	89 614	89 620	89 625	89 631	89 636	89 642	89 647
788	89 653	89 658	89 664	89 669	89 675	89 680	89 686	89 691	89 697	89 702
789	89 708	89 713	89 719	89 724	89 730	89 735	89 741	89 746	89 752	89 757
790	89 763	89 768	89 774	89 779	89 785	89 790	89 796	89 801	89 807	89 812
791	89 818	89 823	89 829	89 834	89 840	89 845	89 851	89 856	89 862	89 867
792	89 873	89 878	89 883	89 889	89 894	89 900	89 905	89 911	89 916	89 922
793	89 927	89 933	89 938	89 944	89 949	89 955	89 960	89 966	89 971	89 977
794	89 982	89 988	89 993	89 998	90 004	90 009	90 015	90 020	90 026	90 031
795	90 037	90 042	90 048	90 053	90 059	90 064	90 069	90 075	90 080	90 086
796	90 091	90 097	90 102	90 108	90 113	90 119	90 124	90 129	90 135	90 140
797	90 146	90 151	90 157	90 162	90 168	90 173	90 179	90 184	90 189	90 195
798	90 200	90 206	90 211	90 217	90 222	90 227	90 233	90 238	90 244	90 249
799	90 255	90 260	90 266	90 271	90 276	90 282	90 287	90 293	90 298	90 304
No.	0	1	2	3	4	5	6	7	8	9

No.	0	1	2	3	4	5	6	7	8	9
800	90 309	90 314	90 320	90 325	90 331	90 336	90 342	90 347	90 352	90 358
801	90 363	90 369	90 374	90 380	90 385	90 390	90 396	90 401	90 407	90 412
802	90 417	90 423	90 428	90 434	90 439	90 445	90 450	90 455	90 461	90 466
803	90 472	90 477	90 482	90 488	90 493	90 499	90 504	90 509	90 515	90 520
804	90 526	90 531	90 536	90 542	90 547	90 553	90 558	90 563	90 569	90 574
805	90 580	90 585	90 590	90 596	90 601	90 607	90 612	90 617	90 623	90 628
806	90 634	90 639	90 644	90 650	90 655	90 660	90 666	90 671	90 677	90 682
807	90 687	90 693	90 698	90 703	90 709	90 714	90 720	90 725	90 730	90 736
808	90 741	90 747	90 752	90 757	90 763	90 768	90 773	90 779	90 784	90 789
809	90 795	90 800	90 806	90 811	90 816	90 822	90 827	90 832	90 838	90 843
810	90 849	90 854	90 859	90 865	90 870	90 875	90 881	90 886	90 891	90 897
811	90 902	90 907	90 913	90 918	90 924	90 929	90 934	90 940	90 945	90 950
812	90 956	90 961	90 966	90 972	90 977	90 982	90 988	90 993	90 998	91 004
813	91 009	91 014	91 020	91 025	91 030	91 036	91 041	91 046	91 052	91 057
814	91 062	91 068	91 073	91 078	91 084	91 089	91 094	91 100	91 105	91 110
815	91 116	91 121	91 126	91 132	91 137	91 142	91 148	91 153	91 158	91 164
816	91 169	91 174	91 180	91 185	91 190	91 196	91 201	91 206	91 212	91 217
817	91 222	91 228	91 233	91 238	91 243	91 249	91 254	91 259	91 265	91 270
818	91 275	91 281	91 286	91 291	91 297	91 302	91 307	91 312	91 318	91 323
819	91 328	91 334	91 339	91 344	91 350	91 355	91 360	91 365	91 371	91 376
820	91 381	91 387	91 392	91 397	91 403	91 408	91 413	91 418	91 424	91 429
821	91 434	91 440	91 445	91 450	91 455	91 461	91 466	91 471	91 477	91 482
822	91 487	91 492	91 498	91 503	91 508	91 514	91 519	91 524	91 529	91 535
823	91 540	91 545	91 551	91 556	91 561	91 566	91 572	91 577	91 582	91 587
824	91 593	91 598	91 603	91 609	91 614	91 619	91 624	91 630	91 635	91 640
825	91 645	91 651	91 656	91 661	91 666	91 672	91 677	91 682	91 687	91 693
826	91 698	91 703	91 709	91 714	91 719	91 724	91 730	91 735	91 740	91 745
827	91 751	91 756	91 761	91 766	91 772	91 777	91 782	91 787	91 793	91 798
828	91 803	91 808	91 814	91 819	91 824	91 829	91 834	91 840	91 845	91 850
829	91 855	91 861	91 866	91 871	91 876	91 882	91 887	91 892	91 897	91 903
830	91 908	91 913	91 918	91 924	91 929	91 934	91 939	91 944	91 950	91 955
831	91 960	91 965	91 971	91 976	91 981	91 986	91 991	91 997	92 002	92 007
832	92 012	92 018	92 023	92 028	92 033	92 038	92 044	92 049	92 054	92 059
833	92 065	92 070	92 075	92 080	92 085	92 091	92 096	92 101	92 106	92 111
834	92 117	92 122	92 127	92 132	92 137	92 143	92 148	92 153	92 158	92 163
835	92 169	92 174	92 179	92 184	92 189	92 195	92 200	92 205	92 210	92 215
836	92 221	92 226	92 231	92 236	92 241	92 247	92 252	92 257	92 262	92 267
837	92 273	92 278	92 283	92 288	92 293	92 298	92 304	92 309	92 314	92 319
838	92 324	92 330	92 335	92 340	92 345	92 350	92 355	92 361	92 366	92 371
839	92 376	92 381	92 387	92 392	92 397	92 402	92 407	92 412	92 418	92 423
840	92 428	92 433	92 438	92 443	92 449	92 454	92 459	92 464	92 469	92 474
841	92 480	92 485	92 490	92 495	92 500	92 505	92 511	92 516	92 521	92 526
842	92 531	92 536	92 542	92 547	92 552	92 557	92 562	92 567	92 572	92 578
843	92 583	92 588	92 593	92 598	92 603	92 609	92 614	92 619	92 624	92 629
844	92 634	92 639	92 645	92 650	92 655	92 660	92 665	92 670	92 675	92 681
845	92 686	92 691	92 696	92 701	92 706	92 711	92 716	92 722	92 727	92 732
846	92 737	92 742	92 747	92 752	92 758	92 763	92 768	92 773	92 778	92 783
847	92 788	92 793	92 799	92 804	92 809	92 814	92 819	92 824	92 829	92 834
848	92 840	92 845	92 850	92 855	92 860	92 865	92 870	92 875	92 881	92 886
849	92 891	92 896	92 901	92 906	92 911	92 916	92 921	92 927	92 932	92 937
No.	0	1	2	3	4	5	6	7	8	9

No.	0	1	2	3	4	5	6	7	8	9
850	92 942	92 947	92 952	92 957	92 962	92 967	92 973	92 978	92 983	92 988
851	92 993	92 998	93 003	93 008	93 013	93 018	93 024	93 029	93 034	93 039
852	93 044	93 049	93 054	93 059	93 064	93 069	93 075	93 080	93 085	93 090
853	93 095	93 100	93 105	93 110	93 115	93 120	93 125	93 131	93 136	93 141
854	93 146	93 151	93 156	93 161	93 166	93 171	93 176	93 181	93 186	93 192
855	93 197	93 202	93 207	93 212	93 217	93 222	93 227	93 232	93 237	93 242
856	93 247	93 252	93 258	93 263	93 268	93 273	93 278	93 283	93 288	93 293
857	93 298	93 303	93 308	93 313	93 318	93 323	93 328	93 334	93 339	93 344
858	93 349	93 354	93 359	93 364	93 369	93 374	93 379	93 384	93 389	93 394
859	93 399	93 404	93 409	93 414	93 420	93 425	93 430	93 435	93 440	93 445
860	93 450	93 455	93 460	93 465	93 470	93 475	93 480	93 485	93 490	93 495
861	93 500	93 505	93 510	93 515	93 520	93 526	93 531	93 536	93 541	93 546
862	93 551	93 556	93 561	93 566	93 571	93 576	93 581	93 586	93 591	93 596
863	93 601	93 606	93 611	93 616	93 621	93 626	93 631	93 636	93 641	93 646
864	93 651	93 656	93 661	93 666	93 671	93 676	93 682	93 687	93 692	93 697
865	93 702	93 707	93 712	93 717	93 722	93 727	93 732	93 737	93 742	93 747
866	93 752	93 757	93 762	93 767	93 772	93 777	93 782	93 787	93 792	93 797
867	93 802	93 807	93 812	93 817	93 822	93 827	93 832	93 837	93 842	93 847
868	93 852	93 857	93 862	93 867	93 872	93 877	93 882	93 887	93 892	93 897
869	93 902	93 907	93 912	93 917	93 922	93 927	93 932	93 937	93 942	93 947
870	93 952	93 957	93 962	93 967	93 972	93 977	93 982	93 987	93 992	93 997
871	94 002	94 007	94 012	94 017	94 022	94 027	94 032	94 037	94 042	94 047
872	94 052	94 057	94 062	94 067	94 072	94 077	94 082	94 086	94 091	94 096
873	94 101	94 106	94 111	94 116	94 121	94 126	94 131	94 136	94 141	94 146
874	94 151	94 156	94 161	94 166	94 171	94 176	94 181	94 186	94 191	94 196
875	94 201	94 206	94 211	94 216	94 221	94 226	94 231	94 236	94 240	94 245
876	94 250	94 255	94 260	94 265	94 270	94 275	94 280	94 285	94 290	94 295
877	94 300	94 305	94 310	94 315	94 320	94 325	94 330	94 335	94 340	94 345
878	94 349	94 354	94 359	94 364	94 369	94 374	94 379	94 384	94 389	94 394
879	94 399	94 404	94 409	94 414	94 419	94 424	94 429	94 433	94 438	94 443
880	94 448	94 453	94 458	94 463	94 468	94 473	94 478	94 483	94 488	94 493
881	94 498	94 503	94 507	94 512	94 517	94 522	94 527	94 532	94 537	94 542
882	94 547	94 552	94 557	94 562	94 567	94 571	94 576	94 581	94 586	94 591
883	94 596	94 601	94 606	94 611	94 616	94 621	94 626	94 630	94 635	94 640
884	94 645	94 650	94 655	94 660	94 665	94 670	94 675	94 680	94 685	94 689
885	94 694	94 699	94 704	94 709	94 714	94 719	94 724	94 729	94 734	94 738
886	94 743	94 748	94 753	94 758	94 763	94 768	94 773	94 778	94 783	94 787
887	94 792	94 797	94 802	94 807	94 812	94 817	94 822	94 827	94 832	94 836
888	94 841	94 846	94 851	94 856	94 861	94 866	94 871	94 876	94 880	94 885
889	94 890	94 895	94 900	94 905	94 910	94 915	94 919	94 924	94 929	94 934
890	94 939	94 944	94 949	94 954	94 959	94 963	94 968	94 973	94 978	94 983
891	94 988	94 993	94 998	95 002	95 007	95 012	95 017	95 022	95 027	95 032
892	95 036	95 041	95 046	95 051	95 056	95 061	95 066	95 071	95 075	95 080
893	95 085	95 090	95 095	95 100	95 105	95 109	95 114	95 119	95 124	95 129
894	95 134	95 139	95 143	95 148	95 153	95 158	95 163	95 168	95 173	95 177
895	95 182	95 187	95 192	95 197	95 202	95 207	95 211	95 216	95 221	95 226
896	95 231	95 236	95 240	95 245	95 250	95 255	95 260	95 265	95 270	95 274
897	95 279	95 284	95 289	95 294	95 299	95 303	95 308	95 313	95 318	95 323
898	95 328	95 332	95 337	95 342	95 347	95 352	95 357	95 361	95 366	95 371
899	95 376	95 381	95 386	95 390	95 395	95 400	95 405	95 410	95 415	95 419
No.	0	1	2	3	4	5	6	7	8	9

No.	0	1	2	3	4	5	6	7	8	9
900	95 424	95 429	95 434	95 439	95 444	95 448	95 453	95 458	95 463	95 468
901	95 472	95 477	95 482	95 487	95 492	95 497	95 501	95 506	95 511	95 516
902	95 521	95 525	95 530	95 535	95 540	95 545	95 550	95 554	95 559	95 564
903	95 569	95 574	95 578	95 583	95 588	95 593	95 598	95 602	95 607	95 612
904	95 617	95 622	95 626	95 631	95 636	95 641	95 646	95 650	95 655	95 660
905	95 665	95 670	95 674	95 679	95 684	95 689	95 694	95 698	95 703	95 708
906	95 713	95 718	95 722	95 727	95 732	95 737	95 742	95 746	95 751	95 756
907	95 761	95 766	95 770	95 775	95 780	95 785	95 789	95 794	95 799	95 804
908	95 809	95 813	95 818	95 823	95 828	95 832	95 837	95 842	95 847	95 852
909	95 856	95 861	95 866	95 871	95 875	95 880	95 885	95 890	95 895	95 899
910	95 904	95 909	95 914	95 918	95 923	95 928	95 933	95 938	95 942	95 947
911	95 952	95 957	95 961	95 966	95 971	95 976	95 980	95 985	95 990	95 995
912	95 999	96 004	96 009	96 014	96 019	96 023	96 028	96 033	96 038	96 042
913	96 047	96 052	96 057	96 061	96 066	96 071	96 076	96 080	96 085	96 090
914	96 095	96 099	96 104	96 109	96 114	96 118	96 123	96 128	96 133	96 137
915	96 142	96 147	96 152	96 156	96 161	96 166	96 171	96 175	96 180	96 185
916	96 190	96 194	96 199	96 204	96 209	96 213	96 218	96 223	96 227	96 232
917	96 237	96 242	96 246	96 251	96 256	96 261	96 265	96 270	96 275	96 280
918	96 284	96 289	96 294	96 298	96 303	96 308	96 313	96 317	96 322	96 327
919	96 332	96 336	96 341	96 346	96 350	96 355	96 360	96 365	96 369	96 374
920	96 379	96 384	96 388	96 393	96 398	96 402	96 407	96 412	96 417	96 421
921	96 426	96 431	96 435	96 440	96 445	96 450	96 454	96 459	96 464	96 468
922	96 473	96 478	96 483	96 487	96 492	96 497	96 501	96 506	96 511	96 515
923	96 520	96 525	96 530	96 534	96 539	96 544	96 548	96 553	96 558	96 562
924	96 567	96 572	96 577	96 581	96 586	96 591	96 595	96 600	96 605	96 609
925	96 614	96 619	96 624	96 628	96 633	96 638	96 642	96 647	96 652	96 656
926	96 661	96 666	96 670	96 675	96 680	96 685	96 689	96 694	96 699	96 703
927	96 708	96 713	96 717	96 722	96 727	96 731	96 736	96 741	96 745	96 750
928	96 755	96 759	96 764	96 769	96 774	96 778	96 783	96 788	96 792	96 797
929	96 802	96 806	96 811	96 816	96 820	96 825	96 830	96 834	96 839	96 844
930	96 848	96 853	96 858	96 862	96 867	96 872	96 876	96 881	96 886	96 890
931	96 895	96 900	96 904	96 909	96 914	96 918	96 923	96 928	96 932	96 937
932	96 942	96 946	96 951	96 956	96 960	96 965	96 970	96 974	96 979	96 984
933	96 988	96 993	96 997	97 002	97 007	97 011	97 016	97 021	97 025	97 030
934	97 035	97 039	97 044	97 049	97 053	97 058	97 063	97 067	97 072	97 077
935	97 081	97 086	97 090	97 095	97 100	97 104	97 109	97 114	97 118	97 123
936	97 128	97 132	97 137	97 142	97 146	97 151	97 155	97 160	97 165	97 169
937	97 174	97 179	97 183	97 188	97 192	97 197	97 202	97 206	97 211	97 216
938	97 220	97 225	97 230	97 234	97 239	97 243	97 248	97 253	97 257	97 262
939	97 267	97 271	97 276	97 280	97 285	97 290	97 294	97 299	97 304	97 308
940	97 313	97 317	97 322	97 327	97 331	97 336	97 340	97 345	97 350	97 354
941	97 359	97 364	97 368	97 373	97 377	97 382	97 387	97 391	97 396	97 400
942	97 405	97 410	97 414	97 419	97 424	97 428	97 433	97 437	97 442	97 447
943	97 451	97 456	97 460	97 465	97 470	97 474	97 479	97 483	97 488	97 493
944	97 497	97 502	97 506	97 511	97 516	97 520	97 525	97 529	97 534	97 539
945	97 543	97 548	97 552	97 557	97 562	97 566	97 571	97 575	97 580	97 585
946	97 589	97 594	97 598	97 603	97 607	97 612	97 617	97 621	97 626	97 630
947	97 635	97 640	97 644	97 649	97 653	97 658	97 663	97 667	97 672	97 676
948	97 681	97 685	97 690	97 695	97 699	97 704	97 708	97 713	97 717	97 722
949	97 727	97 731	97 736	97 740	97 745	97 749	97 754	97 759	97 763	97 768
No.	0	1	2	3	4	5	6	7	8	9

No.	0	1	2	3	4	5	6	7	8	9
950	97 772	97 777	97 782	97 786	97 791	97 795	97 800	97 804	97 809	97 813
951	97 818	97 823	97 827	97 832	97 836	97 841	97 845	97 850	97 855	97 859
952	97 864	97 868	97 873	97 877	97 882	97 886	97 891	97 896	97 900	97 905
953	97 909	97 914	97 918	97 923	97 928	97 932	97 937	97 941	97 946	97 950
954	97 955	97 959	97 964	97 968	97 973	97 978	97 982	97 987	97 991	97 996
955	98 000	98 005	98 009	98 014	98 019	98 023	98 028	98 032	98 037	98 041
956	98 046	98 050	98 055	98 059	98 064	98 068	98 073	98 078	98 082	98 087
957	98 091	98 096	98 100	98 105	98 109	98 114	98 118	98 123	98 127	98 132
958	98 137	98 141	98 146	98 150	98 155	98 159	98 164	98 168	98 173	98 177
959	98 182	98 186	98 191	98 195	98 200	98 204	98 209	98 214	98 218	98 223
960	98 227	98 232	98 236	98 241	98 245	98 250	98 254	98 259	98 263	98 268
961	98 272	98 277	98 281	98 286	98 290	98 295	98 299	98 304	98 308	98 313
962	98 318	98 322	98 327	98 331	98 336	98 340	98 345	98 349	98 354	98 358
963	98 363	98 367	98 372	98 376	98 381	98 385	98 390	98 394	98 399	98 403
964	98 408	98 412	98 417	98 421	98 426	98 430	98 435	98 439	98 444	98 448
965	98 453	98 457	98 462	98 466	98 471	98 475	98 480	98 484	98 489	98 493
966	98 498	98 502	98 507	98 511	98 516	98 520	98 525	98 529	98 534	98 538
967	98 543	98 547	98 552	98 556	98 561	98 565	98 570	98 574	98 579	98 583
968	98 588	98 592	98 597	98 601	98 605	98 610	98 614	98 619	98 623	98 628
969	98 632	98 637	98 641	98 646	98 650	98 655	98 659	98 664	98 668	98 673
970	98 677	98 682	98 686	98 691	98 695	98 700	98 704	98 709	98 713	98 717
971	98 722	98 726	98 731	98 735	98 740	98 744	98 749	98 753	98 758	98 762
972	98 767	98 771	98 776	98 780	98 784	98 789	98 793	98 798	98 802	98 807
973	98 811	98 816	98 820	98 825	98 829	98 834	98 838	98 843	98 847	98 851
974	98 856	98 860	98 865	98 869	98 874	98 878	98 883	98 887	98 892	98 896
975	98 900	98 905	98 909	98 914	98 918	98 923	98 927	98 932	98 936	98 941
976	98 945	98 949	98 954	98 958	98 963	98 967	98 972	98 976	98 981	98 985
977	98 989	98 994	98 998	99 003	99 007	99 012	99 016	99 021	99 025	99 029
978	99 034	99 038	99 043	99 047	99 052	99 056	99 061	99 065	99 069	99 074
979	99 078	99 083	99 087	99 092	99 096	99 100	99 105	99 109	99 114	99 118
980	99 123	99 127	99 131	99 136	99 140	99 145	99 149	99 154	99 158	99 162
981	99 167	99 171	99 176	99 180	99 185	99 189	99 193	99 198	99 202	99 207
982	99 211	99 216	99 220	99 224	99 229	99 233	99 238	99 242	99 247	99 251
983	99 255	99 260	99 264	99 269	99 273	99 277	99 282	99 286	99 291	99 295
984	99 300	99 304	99 308	99 313	99 317	99 322	99 326	99 330	99 335	99 339
985	99 344	99 348	99 352	99 357	99 361	99 366	99 370	99 374	99 379	99 383
986	99 388	99 392	99 396	99 401	99 405	99 410	99 414	99 419	99 423	99 427
987	99 432	99 436	99 441	99 445	99 449	99 454	99 458	99 463	99 467	99 471
988	99 476	99 480	99 484	99 489	99 493	99 498	99 502	99 506	99 511	99 515
989	99 520	99 524	99 528	99 533	99 537	99 542	99 546	99 550	99 555	99 559
990	99 564	99 568	99 572	99 577	99 581	99 585	99 590	99 594	99 599	99 603
991	99 607	99 612	99 616	99 621	99 625	99 629	99 634	99 638	99 642	99 647
992	99 651	99 656	99 660	99 664	99 669	99 673	99 677	99 682	99 686	99 691
993	99 695	99 699	99 704	99 708	99 712	99 717	99 721	99 726	99 730	99 734
994	99 739	99 743	99 747	99 752	99 756	99 760	99 765	99 769	99 774	99 778
995	99 782	99 787	99 791	99 795	99 800	99 804	99 808	99 813	99 817	99 822
996	99 826	99 830	99 835	99 839	99 843	99 848	99 852	99 856	99 861	99 865
997	99 870	99 874	99 878	99 883	99 887	99 891	99 896	99 900	99 904	99 909
998	99 913	99 917	99 922	99 926	99 930	99 935	99 939	99 944	99 948	99 952
999	99 957	99 961	99 965	99 970	99 974	99 978	99 983	99 987	99 991	99 996
1000	00 000	00 004	00 009	00 013	00 017	00 022	00 026	00 030	00 035	00 039
No.	0	1	2	3	4	5	6	7	8	9

'	Log sin	Log tan	Log cot	Log cos	'
0	—	—	—	10.00 000	60
1	6.46 373	6.46 373	13.53 627	10.00 000	59
2	6.76 476	6.76 476	13.23 524	10.00 000	58
3	6.94 085	6.94 085	13.05 915	10.00 000	57
4	7.06 579	7.06 579	12.93 421	10.00 000	56
5	7.16 270	7.16 270	12.83 730	10.00 000	55
6	7.24 188	7.24 188	12.75 812	10.00 000	54
7	7.30 882	7.30 882	12.69 118	10.00 000	53
8	7.36 682	7.36 682	12.63 318	10.00 000	52
9	7.41 797	7.41 797	12.58 203	10.00 000	51
10	7.46 373	7.46 373	12.53 627	10.00 000	50
11	7.50 512	7.50 512	12.49 488	10.00 000	49
12	7.54 291	7.54 291	12.45 709	10.00 000	48
13	7.57 767	7.57 767	12.42 233	10.00 000	47
14	7.60 985	7.60 986	12.39 014	10.00 000	46
15	7.63 982	7.63 982	12.36 018	10.00 000	45
16	7.66 784	7.66 785	12.33 215	10.00 000	44
17	7.69 417	7.69 418	12.30 582	9.99 999	43
18	7.71 900	7.71 900	12.28 100	9.99 999	42
19	7.74 248	7.74 248	12.25 752	9.99 999	41
20	7.76 475	7.76 476	12.23 524	9.99 999	40
21	7.78 594	7.78 595	12.21 405	9.99 999	39
22	7.80 615	7.80 615	12.19 385	9.99 999	38
23	7.82 545	7.82 546	12.17 454	9.99 999	37
24	7.84 393	7.84 394	12.15 606	9.99 999	36
25	7.86 166	7.86 167	12.13 833	9.99 999	35
26	7.87 870	7.87 871	12.12 129	9.99 999	34
27	7.89 509	7.89 510	12.10 490	9.99 999	33
28	7.91 088	7.91 089	12.08 911	9.99 999	32
29	7.92 612	7.92 613	12.07 387	9.99 998	31
30	7.94 084	7.94 086	12.05 914	9.99 998	30
31	7.95 508	7.95 510	12.04 490	9.99 998	29
32	7.96 887	7.96 889	12.03 111	9.99 998	28
33	7.98 223	7.98 225	12.01 775	9.99 998	27
34	7.99 520	7.99 522	12.00 478	9.99 998	26
35	8.00 779	8.00 781	11.99 219	9.99 998	25
36	8.02 002	8.02 004	11.97 996	9.99 998	24
37	8.03 192	8.03 194	11.96 806	9.99 997	23
38	8.04 350	8.04 353	11.95 647	9.99 997	22
39	8.05 478	8.05 481	11.94 519	9.99 997	21
40	8.06 578	8.06 581	11.93 419	9.99 997	20
41	8.07 650	8.07 653	11.92 347	9.99 997	19
42	8.08 696	8.08 700	11.91 300	9.99 997	18
43	8.09 718	8.09 722	11.90 278	9.99 997	17
44	8.10 717	8.10 720	11.89 280	9.99 996	16
45	8.11 693	8.11 696	11.88 304	9.99 996	15
46	8.12 647	8.12 651	11.87 349	9.99 996	14
47	8.13 581	8.13 585	11.86 415	9.99 996	13
48	8.14 495	8.14 500	11.85 500	9.99 996	12
49	8.15 391	8.15 395	11.84 605	9.99 996	11
50	8.16 268	8.16 273	11.83 727	9.99 995	10
51	8.17 128	8.17 135	11.82 867	9.99 995	9
52	8.17 971	8.17 976	11.82 024	9.99 995	8
53	8.18 798	8.18 804	11.81 196	9.99 995	7
54	8.19 610	8.19 616	11.80 384	9.99 995	6
55	8.20 407	8.20 413	11.79 587	9.99 994	5
56	8.21 189	8.21 195	11.78 805	9.99 994	4
57	8.21 958	8.21 964	11.78 036	9.99 994	3
58	8.22 713	8.22 720	11.77 280	9.99 994	2
59	8.23 456	8.23 462	11.76 538	9.99 994	1
60	8.24 186	8.24 192	11.75 808	9.99 993	0
'	Log cos	Log cot	Log tan	Log sin	'

/	8Lsin	8Ltan	11Lcot	9Lcos	/
0	.24 186	.24 192	.75 808	.99 993	60
1	.24 903	.24 910	.75 090	.99 993	59
2	.25 609	.25 616	.74 384	.99 993	58
3	.26 304	.26 312	.73 688	.99 993	57
4	.26 988	.26 996	.73 004	.99 992	56
5	.27 661	.27 669	.72 331	.99 992	55
6	.28 324	.28 332	.71 668	.99 992	54
7	.28 977	.28 986	.71 014	.99 992	53
8	.29 621	.29 629	.70 371	.99 992	52
9	.30 255	.30 263	.69 737	.99 991	51
10	.30 879	.30 888	.69 112	.99 991	50
11	.31 495	.31 505	.68 495	.99 991	49
12	.32 103	.32 112	.67 888	.99 990	48
13	.32 702	.32 711	.67 289	.99 990	47
14	.33 292	.33 302	.66 698	.99 990	46
15	.33 875	.33 886	.66 114	.99 990	45
16	.34 450	.34 461	.65 539	.99 989	44
17	.35 018	.35 029	.64 971	.99 989	43
18	.35 578	.35 590	.64 410	.99 989	42
19	.36 131	.36 143	.63 857	.99 989	41
20	.36 678	.36 689	.63 311	.99 988	40
21	.37 217	.37 229	.62 771	.99 988	39
22	.37 750	.37 762	.62 238	.99 988	38
23	.38 276	.38 289	.61 711	.99 987	37
24	.38 796	.38 809	.61 191	.99 987	36
25	.39 310	.39 323	.60 677	.99 987	35
26	.39 818	.39 832	.60 168	.99 986	34
27	.40 320	.40 334	.59 666	.99 986	33
28	.40 816	.40 830	.59 170	.99 986	32
29	.41 307	.41 321	.58 679	.99 985	31
30	.41 792	.41 807	.58 193	.99 985	30
31	.42 272	.42 287	.57 713	.99 985	29
32	.42 746	.42 762	.57 238	.99 984	28
33	.43 216	.43 232	.56 768	.99 984	27
34	.43 680	.43 696	.56 304	.99 984	26
35	.44 139	.44 156	.55 844	.99 983	25
36	.44 594	.44 611	.55 389	.99 983	24
37	.45 044	.45 061	.54 939	.99 983	23
38	.45 489	.45 507	.54 493	.99 982	22
39	.45 930	.45 948	.54 052	.99 982	21
40	.46 366	.46 385	.53 615	.99 982	20
41	.46 799	.46 817	.53 183	.99 981	19
42	.47 226	.47 245	.52 755	.99 981	18
43	.47 650	.47 669	.52 331	.99 981	17
44	.48 069	.48 089	.51 911	.99 980	16
45	.48 485	.48 505	.51 495	.99 980	15
46	.48 896	.48 917	.51 083	.99 979	14
47	.49 304	.49 325	.50 675	.99 979	13
48	.49 708	.49 729	.50 271	.99 979	12
49	.50 108	.50 130	.49 870	.99 978	11
50	.50 504	.50 527	.49 473	.99 978	10
51	.50 897	.50 920	.49 080	.99 977	9
52	.51 287	.51 310	.48 690	.99 977	8
53	.51 673	.51 696	.48 304	.99 977	7
54	.52 055	.52 079	.47 921	.99 976	6
55	.52 434	.52 459	.47 541	.99 976	5
56	.52 810	.52 835	.47 165	.99 975	4
57	.53 183	.53 208	.46 792	.99 975	3
58	.53 552	.53 578	.46 422	.99 974	2
59	.53 919	.53 945	.46 055	.99 974	1
60	.54 282	.54 308	.45 692	.99 974	0
/	8Lcos	8Lcot	11Ltan	9Lsin	/

/	8Lsin	8Ltan	11Lcot	9Lcos	/
0	.54 282	.54 308	.45 692	.99 974	60
1	.54 642	.54 669	.45 331	.99 973	59
2	.54 999	.55 027	.44 973	.99 973	58
3	.55 354	.55 382	.44 618	.99 972	57
4	.55 705	.55 734	.44 266	.99 972	56
5	.56 054	.56 083	.43 917	.99 971	55
6	.56 400	.56 429	.43 571	.99 971	54
7	.56 743	.56 773	.43 227	.99 970	53
8	.57 084	.57 114	.42 886	.99 970	52
9	.57 421	.57 452	.42 548	.99 969	51
10	.57 757	.57 788	.42 212	.99 969	50
11	.58 089	.58 121	.41 879	.99 968	49
12	.58 419	.58 451	.41 549	.99 968	48
13	.58 747	.58 779	.41 221	.99 967	47
14	.59 072	.59 105	.40 895	.99 967	46
15	.59 395	.59 428	.40 572	.99 967	45
16	.59 715	.59 749	.40 251	.99 966	44
17	.60 033	.60 068	.39 932	.99 966	43
18	.60 349	.60 384	.39 616	.99 965	42
19	.60 662	.60 698	.39 302	.99 964	41
20	.60 973	.61 009	.38 991	.99 964	40
21	.61 282	.61 319	.38 681	.99 963	39
22	.61 589	.61 626	.38 374	.99 963	38
23	.61 894	.61 931	.38 069	.99 962	37
24	.62 196	.62 234	.37 766	.99 962	36
25	.62 497	.62 535	.37 465	.99 961	35
26	.62 795	.62 834	.37 166	.99 961	34
27	.63 091	.63 131	.36 869	.99 960	33
28	.63 385	.63 426	.36 574	.99 960	32
29	.63 678	.63 718	.36 282	.99 959	31
30	.63 968	.64 009	.35 991	.99 959	30
31	.64 256	.64 298	.35 702	.99 958	29
32	.64 543	.64 585	.35 415	.99 958	28
33	.64 827	.64 870	.35 130	.99 957	27
34	.65 110	.65 154	.34 846	.99 956	26
35	.65 391	.65 435	.34 565	.99 956	25
36	.65 670	.65 715	.34 285	.99 955	24
37	.65 947	.65 993	.34 007	.99 955	23
38	.66 223	.66 269	.33 731	.99 954	22
39	.66 497	.66 543	.33 457	.99 954	21
40	.66 769	.66 816	.33 184	.99 953	20
41	.67 039	.67 087	.32 913	.99 952	19
42	.67 308	.67 356	.32 644	.99 952	18
43	.67 575	.67 624	.32 376	.99 951	17
44	.67 841	.67 890	.32 110	.99 951	16
45	.68 104	.68 154	.31 846	.99 950	15
46	.68 367	.68 417	.31 583	.99 949	14
47	.68 627	.68 678	.31 322	.99 949	13
48	.68 886	.68 938	.31 062	.99 948	12
49	.69 144	.69 196	.30 804	.99 948	11
50	.69 400	.69 453	.30 547	.99 947	10
51	.69 654	.69 708	.30 292	.99 946	9
52	.69 907	.69 962	.30 038	.99 946	8
53	.70 159	.70 214	.29 786	.99 945	7
54	.70 409	.70 465	.29 535	.99 944	6
55	.70 658	.70 714	.29 286	.99 944	5
56	.70 905	.70 962	.29 038	.99 943	4
57	.71 151	.71 208	.28 792	.99 942	3
58	.71 395	.71 453	.28 547	.99 942	2
59	.71 638	.71 697	.28 303	.99 941	1
60	.71 880	.71 940	.28 060	.99 940	0
/	8Lcos	8Lcot	11Ltan	9Lsin	/

/	8 Lsin	8 Ltan	11 Lcot	9 Lcos	/
0	.71880	.71940	.28060	.99940	60
1	.72120	.72181	.27819	.99940	59
2	.72359	.72420	.27580	.99939	58
3	.72597	.72659	.27341	.99938	57
4	.72834	.72896	.27104	.99938	56
5	.73069	.73132	.26868	.99937	55
6	.73303	.73366	.26634	.99936	54
7	.73535	.73600	.26400	.99936	53
8	.73767	.73832	.26168	.99935	52
9	.73997	.74063	.25937	.99934	51
10	.74226	.74292	.25708	.99934	50
11	.74454	.74521	.25479	.99933	49
12	.74680	.74748	.25252	.99932	48
13	.74906	.74974	.25026	.99932	47
14	.75130	.75199	.24801	.99931	46
15	.75353	.75423	.24577	.99930	45
16	.75575	.75645	.24355	.99929	44
17	.75795	.75867	.24133	.99929	43
18	.76015	.76087	.23913	.99928	42
19	.76234	.76306	.23694	.99927	41
20	.76451	.76525	.23475	.99926	40
21	.76667	.76742	.23258	.99926	39
22	.76883	.76958	.23042	.99925	38
23	.77097	.77173	.22827	.99924	37
24	.77310	.77387	.22613	.99923	36
25	.77522	.77600	.22400	.99923	35
26	.77733	.77811	.22189	.99922	34
27	.77943	.78022	.21978	.99921	33
28	.78152	.78232	.21768	.99920	32
29	.78360	.78441	.21559	.99920	31
30	.78568	.78649	.21351	.99919	30
31	.78774	.78855	.21145	.99918	29
32	.78979	.79061	.20939	.99917	28
33	.79183	.79266	.20734	.99917	27
34	.79386	.79470	.20530	.99916	26
35	.79588	.79673	.20327	.99915	25
36	.79789	.79875	.20125	.99914	24
37	.79990	.80076	.19924	.99913	23
38	.80189	.80277	.19723	.99913	22
39	.80388	.80476	.19524	.99912	21
40	.80585	.80674	.19326	.99911	20
41	.80782	.80872	.19128	.99910	19
42	.80978	.81068	.18932	.99909	18
43	.81173	.81264	.18736	.99909	17
44	.81367	.81459	.18541	.99908	16
45	.81560	.81653	.18347	.99907	15
46	.81752	.81846	.18154	.99906	14
47	.81944	.82038	.17962	.99905	13
48	.82134	.82230	.17770	.99904	12
49	.82324	.82420	.17580	.99904	11
50	.82513	.82610	.17390	.99903	10
51	.82701	.82799	.17201	.99902	9
52	.82888	.82987	.17013	.99901	8
53	.83075	.83175	.16825	.99900	7
54	.83261	.83361	.16639	.99899	6
55	.83446	.83547	.16453	.99898	5
56	.83630	.83732	.16268	.99898	4
57	.83813	.83916	.16084	.99897	3
58	.83996	.84100	.15900	.99896	2
59	.84177	.84282	.15718	.99895	1
60	.84358	.84464	.15536	.99894	0
/	8 Lcos	8 Lcot	11 Ltan	9 Lsin	/

/	8 Lsin	8 Ltan	11 Lcot	9 Lcos	/
0	.84358	.84464	.15536	.99894	60
1	.84539	.84646	.15354	.99893	59
2	.84718	.84826	.15174	.99892	58
3	.84897	.85006	.14994	.99891	57
4	.85075	.85185	.14815	.99891	56
5	.85252	.85363	.14637	.99890	55
6	.85429	.85540	.14460	.99889	54
7	.85605	.85717	.14283	.99888	53
8	.85780	.85893	.14107	.99887	52
9	.85955	.86069	.13931	.99886	51
10	.86128	.86243	.13757	.99885	50
11	.86301	.86417	.13583	.99884	49
12	.86474	.86591	.13409	.99883	48
13	.86645	.86763	.13237	.99882	47
14	.86816	.86935	.13065	.99881	46
15	.86987	.87106	.12894	.99880	45
16	.87156	.87277	.12723	.99879	44
17	.87325	.87447	.12553	.99879	43
18	.87494	.87616	.12384	.99878	42
19	.87661	.87785	.12215	.99877	41
20	.87829	.87953	.12047	.99876	40
21	.87995	.88120	.11880	.99875	39
22	.88161	.88287	.11713	.99874	38
23	.88326	.88453	.11547	.99873	37
24	.88490	.88618	.11382	.99872	36
25	.88654	.88783	.11217	.99871	35
26	.88817	.88948	.11052	.99870	34
27	.88980	.89111	.10889	.99869	33
28	.89142	.89274	.10726	.99868	32
29	.89304	.89437	.10563	.99867	31
30	.89464	.89598	.10402	.99866	30
31	.89625	.89760	.10240	.99865	29
32	.89784	.89920	.10080	.99864	28
33	.89943	.90080	.99920	.99863	27
34	.90102	.90240	.99760	.99862	26
35	.90260	.90399	.99601	.99861	25
36	.90417	.90557	.99443	.99860	24
37	.90574	.90715	.99285	.99859	23
38	.90730	.90872	.99128	.99858	22
39	.90885	.91029	.98971	.99857	21
40	.91040	.91185	.98815	.99856	20
41	.91195	.91340	.98660	.99855	19
42	.91349	.91495	.98505	.99854	18
43	.91502	.91650	.98350	.99853	17
44	.91655	.91803	.98197	.99852	16
45	.91807	.91957	.98043	.99851	15
46	.91959	.92110	.97890	.99850	14
47	.92110	.92262	.97738	.99849	13
48	.92261	.92414	.97586	.99847	12
49	.92411	.92565	.97435	.99846	11
50	.92561	.92716	.97284	.99845	10
51	.92710	.92866	.97134	.99844	9
52	.92859	.93016	.96984	.99843	8
53	.93007	.93165	.96835	.99842	7
54	.93154	.93313	.96687	.99841	6
55	.93301	.93462	.96538	.99840	5
56	.93448	.93609	.96391	.99839	4
57	.93594	.93756	.96244	.99838	3
58	.93740	.93903	.96097	.99837	2
59	.93885	.94049	.95951	.99836	1
60	.94030	.94195	.95805	.99834	0
/	8 Lcos	8 Lcot	11 Ltan	9 Lsin	/

5°

/	8 L sin	8 L tan	11 L cot	9 L cos	/
0	.94 030	.94 195	.05 805	.99 834	60
1	.94 174	.94 340	.05 660	.99 833	59
2	.94 317	.94 485	.05 515	.99 832	58
3	.94 461	.94 630	.05 370	.99 831	57
4	.94 603	.94 773	.05 227	.99 830	56
5	.94 746	.94 917	.05 083	.99 829	55
6	.94 887	.95 060	.04 940	.99 828	54
7	.95 029	.95 202	.04 798	.99 827	53
8	.95 170	.95 344	.04 656	.99 825	52
9	.95 310	.95 486	.04 514	.99 824	51
10	.95 450	.95 627	.04 373	.99 823	50
11	.95 589	.95 767	.04 233	.99 822	49
12	.95 728	.95 908	.04 092	.99 821	48
13	.95 867	.96 047	.03 953	.99 820	47
14	.96 005	.96 187	.03 813	.99 819	46
15	.96 143	.96 325	.03 675	.99 817	45
16	.96 280	.96 464	.03 536	.99 816	44
17	.96 417	.96 602	.03 398	.99 815	43
18	.96 553	.96 739	.03 261	.99 814	42
19	.96 689	.96 877	.03 123	.99 813	41
20	.96 825	.97 013	.02 987	.99 812	40
21	.96 960	.97 150	.02 850	.99 810	39
22	.97 095	.97 285	.02 715	.99 809	38
23	.97 229	.97 421	.02 579	.99 808	37
24	.97 363	.97 556	.02 444	.99 807	36
25	.97 496	.97 691	.02 309	.99 806	35
26	.97 629	.97 825	.02 175	.99 804	34
27	.97 762	.97 959	.02 041	.99 803	33
28	.97 894	.98 092	.01 908	.99 802	32
29	.98 026	.98 225	.01 775	.99 801	31
30	.98 157	.98 358	.01 642	.99 800	30
31	.98 288	.98 490	.01 510	.99 798	29
32	.98 419	.98 622	.01 378	.99 797	28
33	.98 549	.98 753	.01 247	.99 796	27
34	.98 679	.98 884	.01 116	.99 795	26
35	.98 808	.99 015	.00 985	.99 793	25
36	.98 937	.99 145	.00 855	.99 792	24
37	.99 066	.99 275	.00 725	.99 791	23
38	.99 194	.99 405	.00 595	.99 790	22
39	.99 322	.99 534	.00 466	.99 788	21
40	.99 450	.99 662	.00 338	.99 787	20
41	.99 577	.99 791	.00 209	.99 786	19
42	.99 704	.99 919	.00 081	.99 785	18
43	.99 830	.00 046	.99 954	.99 783	17
44	.99 956	.00 174	.99 826	.99 782	16
45	.00 082	.00 301	.99 699	.99 781	15
46	.00 207	.00 427	.99 573	.99 780	14
47	.00 332	.00 553	.99 447	.99 778	13
48	.00 456	.00 679	.99 321	.99 777	12
49	.00 581	.00 805	.99 195	.99 776	11
50	.00 704	.00 930	.99 070	.99 775	10
51	.00 828	.01 055	.98 945	.99 773	9
52	.00 951	.01 179	.98 821	.99 772	8
53	.01 074	.01 303	.98 697	.99 771	7
54	.01 196	.01 427	.98 573	.99 769	6
55	.01 318	.01 550	.98 450	.99 768	5
56	.01 440	.01 673	.98 327	.99 767	4
57	.01 561	.01 796	.98 204	.99 765	3
58	.01 682	.01 918	.98 082	.99 764	2
59	.01 803	.02 040	.97 960	.99 763	1
60	.01 923	.02 162	.97 838	.99 761	0
/	9 L cos	9 L cot	10 L tan	9 L sin	/

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xxiii

/	9 L sin	9 L tan	10 L cot	9 L cos	/
0	.01 923	.02 162	.97 838	.99 761	60
1	.02 043	.02 283	.97 717	.99 760	59
2	.02 163	.02 404	.97 596	.99 759	58
3	.02 283	.02 525	.97 475	.99 757	57
4	.02 402	.02 645	.97 355	.99 756	56
5	.02 520	.02 766	.97 234	.99 755	55
6	.02 639	.02 885	.97 115	.99 753	54
7	.02 757	.03 005	.96 995	.99 752	53
8	.02 874	.03 124	.96 876	.99 751	52
9	.02 992	.03 242	.96 758	.99 749	51
10	.03 109	.03 361	.96 639	.99 748	50
11	.03 226	.03 479	.96 521	.99 747	49
12	.03 342	.03 597	.96 403	.99 745	48
13	.03 458	.03 714	.96 286	.99 744	47
14	.03 574	.03 832	.96 168	.99 742	46
15	.03 690	.03 948	.96 052	.99 741	45
16	.03 805	.04 065	.95 935	.99 740	44
17	.03 920	.04 181	.95 819	.99 738	43
18	.04 034	.04 297	.95 703	.99 737	42
19	.04 149	.04 413	.95 587	.99 736	41
20	.04 262	.04 528	.95 472	.99 734	40
21	.04 376	.04 643	.95 357	.99 733	39
22	.04 490	.04 758	.95 242	.99 731	38
23	.04 603	.04 873	.95 127	.99 730	37
24	.04 715	.04 987	.95 013	.99 728	36
25	.04 828	.05 101	.94 899	.99 727	35
26	.04 940	.05 214	.94 786	.99 726	34
27	.05 052	.05 328	.94 672	.99 724	33
28	.05 164	.05 441	.94 559	.99 723	32
29	.05 275	.05 553	.94 447	.99 721	31
30	.05 386	.05 666	.94 334	.99 720	30
31	.05 497	.05 778	.94 222	.99 718	29
32	.05 607	.05 890	.94 110	.99 717	28
33	.05 717	.06 002	.93 998	.99 716	27
34	.05 827	.06 113	.93 887	.99 714	26
35	.05 937	.06 224	.93 776	.99 713	25
36	.06 046	.06 335	.93 665	.99 711	24
37	.06 155	.06 445	.93 555	.99 710	23
38	.06 264	.06 556	.93 444	.99 708	22
39	.06 372	.06 666	.93 334	.99 707	21
40	.06 481	.06 775	.93 225	.99 705	20
41	.06 589	.06 885	.93 115	.99 704	19
42	.06 696	.06 994	.93 006	.99 702	18
43	.06 804	.07 103	.92 897	.99 701	17
44	.06 911	.07 211	.92 789	.99 699	16
45	.07 018	.07 320	.92 680	.99 698	15
46	.07 124	.07 428	.92 572	.99 696	14
47	.07 231	.07 536	.92 464	.99 695	13
48	.07 337	.07 643	.92 357	.99 693	12
49	.07 442	.07 751	.92 249	.99 692	11
50	.07 548	.07 858	.92 142	.99 690	10
51	.07 653	.07 964	.92 036	.99 689	9
52	.07 758	.08 071	.91 929	.99 687	8
53	.07 863	.08 177	.91 823	.99 686	7
54	.07 968	.08 283	.91 717	.99 684	6
55	.08 072	.08 389	.91 611	.99 683	5
56	.08 176	.08 495	.91 505	.99 681	4
57	.08 280	.08 600	.91 400	.99 680	3
58	.08 383	.08 705	.91 295	.99 678	2
59	.08 486	.08 810	.91 190	.99 677	1
60	.08 589	.08 914	.91 086	.99 675	0
/	9 L cos	9 L cot	10 L tan	9 L sin	/

83°

/	θ Lsin	θ Ltan	θ Lcot	θ Lcos	/
0	.08589	.08914	.91086	.99675	60
1	.08692	.09019	.90981	.99674	59
2	.08795	.09123	.90877	.99672	58
3	.08897	.09227	.90773	.99670	57
4	.08999	.09330	.90670	.99669	56
5	.09101	.09434	.90566	.99667	55
6	.09202	.09537	.90463	.99666	54
7	.09304	.09640	.90360	.99664	53
8	.09405	.09742	.90258	.99663	52
9	.09506	.09845	.90155	.99661	51
10	.09606	.09947	.90053	.99659	50
11	.09707	.10049	.89951	.99658	49
12	.09807	.10150	.89850	.99656	48
13	.09907	.10252	.89748	.99655	47
14	.10006	.10353	.89647	.99653	46
15	.10106	.10454	.89546	.99651	45
16	.10205	.10555	.89445	.99650	44
17	.10304	.10656	.89344	.99648	43
18	.10402	.10756	.89244	.99647	42
19	.10501	.10856	.89144	.99645	41
20	.10599	.10956	.89044	.99643	40
21	.10697	.11056	.88944	.99642	39
22	.10795	.11155	.88845	.99640	38
23	.10893	.11254	.88746	.99638	37
24	.10990	.11353	.88647	.99637	36
25	.11087	.11452	.88548	.99635	35
26	.11184	.11551	.88449	.99633	34
27	.11281	.11649	.88351	.99632	33
28	.11377	.11747	.88253	.99630	32
29	.11474	.11845	.88155	.99629	31
30	.11570	.11943	.88057	.99627	30
31	.11666	.12040	.87960	.99625	29
32	.11761	.12138	.87862	.99624	28
33	.11857	.12235	.87765	.99622	27
34	.11952	.12332	.87668	.99620	26
35	.12047	.12428	.87572	.99618	25
36	.12142	.12525	.87475	.99617	24
37	.12236	.12621	.87379	.99615	23
38	.12331	.12717	.87283	.99613	22
39	.12425	.12813	.87187	.99612	21
40	.12519	.12909	.87091	.99610	20
41	.12612	.13004	.86996	.99608	19
42	.12706	.13099	.86901	.99607	18
43	.12799	.13194	.86806	.99605	17
44	.12892	.13289	.86711	.99603	16
45	.12985	.13384	.86616	.99601	15
46	.13078	.13478	.86522	.99600	14
47	.13171	.13573	.86427	.99598	13
48	.13263	.13667	.86333	.99596	12
49	.13355	.13761	.86239	.99595	11
50	.13447	.13854	.86146	.99593	10
51	.13539	.13948	.86052	.99591	9
52	.13630	.14041	.85959	.99589	8
53	.13722	.14134	.85866	.99588	7
54	.13813	.14227	.85773	.99586	6
55	.13904	.14320	.85680	.99584	5
56	.13994	.14412	.85588	.99582	4
57	.14085	.14504	.85496	.99581	3
58	.14175	.14597	.85403	.99579	2
59	.14266	.14688	.85312	.99577	1
60	.14356	.14780	.85220	.99575	0
/	θ Lcos	θ Lcot	θ Ltan	θ Lsin	/

/	θ Lsin	θ Ltan	θ Lcot	θ Lcos	/
0	.14356	.14780	.85220	.99575	60
1	.14445	.14872	.85128	.99574	59
2	.14535	.14963	.85037	.99572	58
3	.14624	.15054	.84946	.99570	57
4	.14714	.15145	.84855	.99568	56
5	.14803	.15236	.84764	.99566	55
6	.14891	.15327	.84673	.99565	54
7	.14980	.15417	.84583	.99563	53
8	.15069	.15508	.84492	.99561	52
9	.15157	.15598	.84402	.99559	51
10	.15245	.15688	.84312	.99557	50
11	.15333	.15777	.84223	.99556	49
12	.15421	.15867	.84133	.99554	48
13	.15508	.15956	.84044	.99552	47
14	.15596	.16046	.83954	.99550	46
15	.15683	.16135	.83865	.99548	45
16	.15770	.16224	.83776	.99546	44
17	.15857	.16312	.83688	.99545	43
18	.15944	.16401	.83599	.99543	42
19	.16030	.16489	.83511	.99541	41
20	.16116	.16577	.83423	.99539	40
21	.16203	.16665	.83335	.99537	39
22	.16289	.16753	.83247	.99535	38
23	.16374	.16841	.83159	.99533	37
24	.16460	.16928	.83072	.99532	36
25	.16545	.17016	.82984	.99530	35
26	.16631	.17103	.82897	.99528	34
27	.16716	.17190	.82810	.99526	33
28	.16801	.17277	.82723	.99524	32
29	.16886	.17363	.82637	.99522	31
30	.16970	.17450	.82550	.99520	30
31	.17055	.17536	.82464	.99518	29
32	.17139	.17622	.82378	.99517	28
33	.17223	.17708	.82292	.99515	27
34	.17307	.17794	.82206	.99513	26
35	.17391	.17880	.82120	.99511	25
36	.17474	.17965	.82035	.99509	24
37	.17558	.18051	.81949	.99507	23
38	.17641	.18136	.81864	.99505	22
39	.17724	.18221	.81779	.99503	21
40	.17807	.18306	.81694	.99501	20
41	.17890	.18391	.81609	.99499	19
42	.17973	.18475	.81525	.99497	18
43	.18055	.18560	.81440	.99495	17
44	.18137	.18644	.81356	.99494	16
45	.18220	.18728	.81272	.99492	15
46	.18302	.18812	.81188	.99490	14
47	.18383	.18896	.81104	.99488	13
48	.18465	.18979	.81021	.99486	12
49	.18547	.19063	.80937	.99484	11
50	.18628	.19146	.80854	.99482	10
51	.18709	.19229	.80771	.99480	9
52	.18790	.19312	.80688	.99478	8
53	.18871	.19395	.80605	.99476	7
54	.18952	.19478	.80522	.99474	6
55	.19033	.19561	.80439	.99472	5
56	.19113	.19643	.80357	.99470	4
57	.19193	.19725	.80275	.99468	3
58	.19273	.19807	.80193	.99466	2
59	.19353	.19889	.80111	.99464	1
60	.19433	.19971	.80029	.99462	0
/	θ Lcos	θ Lcot	θ Ltan	θ Lsin	/

9°

/	9 L sin	9 L tan	10 L cot	9 L cos	/
0	.19433	.19971	.80029	.99462	60
1	.19513	.20053	.79947	.99460	59
2	.19592	.20134	.79866	.99458	58
3	.19672	.20216	.79784	.99456	57
4	.19751	.20297	.79703	.99454	56
5	.19830	.20378	.79622	.99452	55
6	.19909	.20459	.79541	.99450	54
7	.19988	.20540	.79460	.99448	53
8	.20067	.20621	.79379	.99446	52
9	.20145	.20701	.79299	.99444	51
10	.20223	.20782	.79218	.99442	50
11	.20302	.20862	.79138	.99440	49
12	.20380	.20942	.79058	.99438	48
13	.20458	.21022	.78978	.99436	47
14	.20535	.21102	.78898	.99434	46
15	.20613	.21182	.78818	.99432	45
16	.20691	.21261	.78739	.99429	44
17	.20768	.21341	.78659	.99427	43
18	.20845	.21420	.78580	.99425	42
19	.20922	.21499	.78501	.99423	41
20	.20999	.21578	.78422	.99421	40
21	.21076	.21657	.78343	.99419	39
22	.21153	.21736	.78264	.99417	38
23	.21229	.21814	.78186	.99415	37
24	.21306	.21893	.78107	.99413	36
25	.21382	.21971	.78029	.99411	35
26	.21458	.22049	.77951	.99409	34
27	.21534	.22127	.77873	.99407	33
28	.21610	.22205	.77795	.99404	32
29	.21685	.22283	.77717	.99402	31
30	.21761	.22361	.77639	.99400	30
31	.21836	.22438	.77562	.99398	29
32	.21912	.22516	.77484	.99396	28
33	.21987	.22593	.77407	.99394	27
34	.22062	.22670	.77330	.99392	26
35	.22137	.22747	.77253	.99390	25
36	.22211	.22824	.77176	.99388	24
37	.22286	.22901	.77099	.99385	23
38	.22361	.22977	.77023	.99383	22
39	.22435	.23054	.76946	.99381	21
40	.22509	.23130	.76870	.99379	20
41	.22583	.23206	.76794	.99377	19
42	.22657	.23283	.76717	.99375	18
43	.22731	.23359	.76641	.99372	17
44	.22805	.23435	.76565	.99370	16
45	.22878	.23510	.76490	.99368	15
46	.22952	.23586	.76414	.99366	14
47	.23025	.23661	.76339	.99364	13
48	.23098	.23737	.76263	.99362	12
49	.23171	.23812	.76188	.99359	11
50	.23244	.23887	.76113	.99357	10
51	.23317	.23962	.76038	.99355	9
52	.23390	.24037	.75963	.99353	8
53	.23462	.24112	.75888	.99351	7
54	.23535	.24186	.75814	.99348	6
55	.23607	.24261	.75739	.99346	5
56	.23679	.24335	.75665	.99344	4
57	.23752	.24410	.75590	.99342	3
58	.23823	.24484	.75516	.99340	2
59	.23895	.24558	.75442	.99337	1
60	.23967	.24632	.75368	.99335	0
/	9 L cos	9 L cot	10 L tan	9 L sin	/

80°

10°

XXV

/	9 L sin	9 L tan	10 L cot	9 L cos	/
0	.23967	.24632	.75368	.99335	60
1	.24039	.24706	.75294	.99333	59
2	.24110	.24779	.75221	.99331	58
3	.24181	.24853	.75147	.99328	57
4	.24253	.24926	.75074	.99326	56
5	.24324	.25000	.75000	.99324	55
6	.24395	.25073	.74927	.99322	54
7	.24466	.25146	.74854	.99319	53
8	.24536	.25219	.74781	.99317	52
9	.24607	.25292	.74708	.99315	51
10	.24677	.25365	.74635	.99313	50
11	.24748	.25437	.74563	.99310	49
12	.24818	.25510	.74490	.99308	48
13	.24888	.25582	.74418	.99306	47
14	.24958	.25655	.74345	.99304	46
15	.25028	.25727	.74273	.99301	45
16	.25098	.25799	.74201	.99299	44
17	.25168	.25871	.74129	.99297	43
18	.25237	.25943	.74057	.99294	42
19	.25307	.26015	.73985	.99292	41
20	.25376	.26086	.73914	.99290	40
21	.25445	.26158	.73842	.99288	39
22	.25514	.26229	.73771	.99285	38
23	.25583	.26301	.73699	.99283	37
24	.25652	.26372	.73628	.99281	36
25	.25721	.26443	.73557	.99278	35
26	.25790	.26514	.73486	.99276	34
27	.25858	.26585	.73415	.99274	33
28	.25927	.26655	.73345	.99271	32
29	.25995	.26726	.73274	.99269	31
30	.26063	.26797	.73203	.99267	30
31	.26131	.26867	.73133	.99264	29
32	.26199	.26937	.73063	.99262	28
33	.26267	.27008	.72992	.99260	27
34	.26335	.27078	.72922	.99257	26
35	.26403	.27148	.72852	.99255	25
36	.26470	.27218	.72782	.99252	24
37	.26538	.27288	.72712	.99250	23
38	.26605	.27357	.72643	.99248	22
39	.26672	.27427	.72573	.99245	21
40	.26739	.27496	.72504	.99243	20
41	.26806	.27566	.72434	.99241	19
42	.26873	.27635	.72365	.99238	18
43	.26940	.27704	.72296	.99236	17
44	.27007	.27773	.72227	.99233	16
45	.27073	.27842	.72158	.99231	15
46	.27140	.27911	.72089	.99229	14
47	.27206	.27980	.72020	.99226	13
48	.27273	.28049	.71951	.99224	12
49	.27339	.28117	.71883	.99221	11
50	.27405	.28186	.71814	.99219	10
51	.27471	.28254	.71746	.99217	9
52	.27537	.28323	.71677	.99214	8
53	.27602	.28391	.71609	.99212	7
54	.27668	.28459	.71541	.99209	6
55	.27734	.28527	.71473	.99207	5
56	.27799	.28595	.71405	.99204	4
57	.27864	.28662	.71338	.99202	3
58	.27930	.28730	.71270	.99200	2
59	.27995	.28798	.71202	.99197	1
60	.28060	.28865	.71135	.99195	0
/	9 L cos	9 L cot	10 L tan	9 L sin	/

79°

/	9 Lsin	9 Ltan	10 Lcot	9 Lcos	/
0	.28060	.28865	.71135	.99195	60
1	.28125	.28933	.71067	.99192	59
2	.28190	.29000	.71000	.99190	58
3	.28254	.29067	.70933	.99187	57
4	.28319	.29134	.70866	.99185	56
5	.28384	.29201	.70799	.99182	55
6	.28448	.29268	.70732	.99180	54
7	.28512	.29335	.70665	.99177	53
8	.28577	.29402	.70598	.99175	52
9	.28641	.29468	.70532	.99172	51
10	.28705	.29535	.70465	.99170	50
11	.28769	.29601	.70399	.99167	49
12	.28833	.29668	.70332	.99165	48
13	.28896	.29734	.70266	.99162	47
14	.28960	.29800	.70200	.99160	46
15	.29024	.29866	.70134	.99157	45
16	.29087	.29932	.70068	.99155	44
17	.29150	.29998	.70002	.99152	43
18	.29214	.30064	.69936	.99150	42
19	.29277	.30130	.69870	.99147	41
20	.29340	.30195	.69805	.99145	40
21	.29403	.30261	.69739	.99142	39
22	.29466	.30326	.69674	.99140	38
23	.29529	.30391	.69609	.99137	37
24	.29591	.30457	.69543	.99135	36
25	.29654	.30522	.69478	.99132	35
26	.29716	.30587	.69413	.99130	34
27	.29779	.30652	.69348	.99127	33
28	.29841	.30717	.69283	.99124	32
29	.29903	.30782	.69218	.99122	31
30	.29966	.30846	.69154	.99119	30
31	.30028	.30911	.69089	.99117	29
32	.30090	.30975	.69025	.99114	28
33	.30151	.31040	.68960	.99112	27
34	.30213	.31104	.68896	.99109	26
35	.30275	.31168	.68832	.99106	25
36	.30336	.31233	.68767	.99104	24
37	.30398	.31297	.68703	.99101	23
38	.30459	.31361	.68639	.99099	22
39	.30521	.31425	.68575	.99096	21
40	.30582	.31489	.68511	.99093	20
41	.30643	.31552	.68448	.99091	19
42	.30704	.31616	.68384	.99088	18
43	.30765	.31679	.68321	.99086	17
44	.30826	.31743	.68257	.99083	16
45	.30887	.31806	.68194	.99080	15
46	.30947	.31870	.68130	.99078	14
47	.31008	.31933	.68067	.99075	13
48	.31068	.31996	.68004	.99072	12
49	.31129	.32059	.67941	.99070	11
50	.31189	.32122	.67878	.99067	10
51	.31250	.32185	.67815	.99064	9
52	.31310	.32248	.67752	.99062	8
53	.31370	.32311	.67689	.99059	7
54	.31430	.32373	.67627	.99056	6
55	.31490	.32436	.67564	.99054	5
56	.31549	.32498	.67502	.99051	4
57	.31609	.32561	.67439	.99048	3
58	.31669	.32623	.67377	.99046	2
59	.31728	.32685	.67315	.99043	1
60	.31788	.32747	.67253	.99040	0
/	9 Lcos	9 Lcot	10 Ltan	9 Lsin	/

/	9 Lsin	9 Ltan	10 Lcot	9 Lcos	/
0	.31788	.32747	.67253	.99040	60
1	.31847	.32810	.67190	.99038	59
2	.31907	.32872	.67128	.99035	58
3	.31966	.32933	.67067	.99032	57
4	.32025	.32995	.67005	.99030	56
5	.32084	.33057	.66943	.99027	55
6	.32143	.33119	.66881	.99024	54
7	.32202	.33180	.66820	.99022	53
8	.32261	.33242	.66758	.99019	52
9	.32319	.33303	.66697	.99016	51
10	.32378	.33365	.66635	.99013	50
11	.32437	.33426	.66574	.99011	49
12	.32495	.33487	.66513	.99008	48
13	.32553	.33548	.66452	.99005	47
14	.32612	.33609	.66391	.99002	46
15	.32670	.33670	.66330	.99000	45
16	.32728	.33731	.66269	.98997	44
17	.32786	.33792	.66208	.98994	43
18	.32844	.33853	.66147	.98991	42
19	.32902	.33913	.66087	.98989	41
20	.32960	.33974	.66026	.98986	40
21	.33018	.34034	.65966	.98983	39
22	.33075	.34095	.65905	.98980	38
23	.33133	.34155	.65845	.98978	37
24	.33190	.34215	.65785	.98975	36
25	.33248	.34276	.65724	.98972	35
26	.33305	.34336	.65664	.98969	34
27	.33362	.34396	.65604	.98967	33
28	.33420	.34456	.65544	.98964	32
29	.33477	.34516	.65484	.98961	31
30	.33534	.34576	.65424	.98958	30
31	.33591	.34635	.65365	.98955	29
32	.33647	.34695	.65305	.98953	28
33	.33704	.34755	.65245	.98950	27
34	.33761	.34814	.65186	.98947	26
35	.33818	.34874	.65126	.98944	25
36	.33874	.34933	.65067	.98941	24
37	.33931	.34992	.65008	.98938	23
38	.33987	.35051	.64949	.98936	22
39	.34043	.35111	.64889	.98933	21
40	.34100	.35170	.64830	.98930	20
41	.34156	.35229	.64771	.98927	19
42	.34212	.35288	.64712	.98924	18
43	.34268	.35347	.64653	.98921	17
44	.34324	.35405	.64595	.98919	16
45	.34380	.35464	.64536	.98916	15
46	.34436	.35523	.64477	.98913	14
47	.34491	.35581	.64419	.98910	13
48	.34547	.35640	.64360	.98907	12
49	.34602	.35698	.64302	.98904	11
50	.34658	.35757	.64243	.98901	10
51	.34713	.35815	.64185	.98898	9
52	.34769	.35873	.64127	.98896	8
53	.34824	.35931	.64069	.98893	7
54	.34879	.35989	.64011	.98890	6
55	.34934	.36047	.63953	.98887	5
56	.34989	.36105	.63895	.98884	4
57	.35044	.36163	.63837	.98881	3
58	.35099	.36221	.63779	.98878	2
59	.35154	.36279	.63721	.98875	1
60	.35209	.36336	.63664	.98872	0
/	9 Lcos	9 Lcot	10 Ltan	9 Lsin	/

/	°L sin	°L tan	10 L cot	°L cos	/
0	.35 209	.36 336	.63 664	.98 872	60
1	.35 263	.36 394	.63 606	.98 869	59
2	.35 318	.36 452	.63 548	.98 867	58
3	.35 373	.36 509	.63 491	.98 864	57
4	.35 427	.36 566	.63 434	.98 861	56
5	.35 481	.36 624	.63 376	.98 858	55
6	.35 536	.36 681	.63 319	.98 855	54
7	.35 590	.36 738	.63 262	.98 852	53
8	.35 644	.36 795	.63 205	.98 849	52
9	.35 698	.36 852	.63 148	.98 846	51
10	.35 752	.36 909	.63 091	.98 843	50
11	.35 806	.36 966	.63 034	.98 840	49
12	.35 860	.37 023	.62 977	.98 837	48
13	.35 914	.37 080	.62 920	.98 834	47
14	.35 968	.37 137	.62 863	.98 831	46
15	.36 022	.37 193	.62 807	.98 828	45
16	.36 075	.37 250	.62 750	.98 825	44
17	.36 129	.37 306	.62 694	.98 822	43
18	.36 182	.37 363	.62 637	.98 819	42
19	.36 236	.37 419	.62 581	.98 816	41
20	.36 289	.37 476	.62 524	.98 813	40
21	.36 342	.37 532	.62 468	.98 810	39
22	.36 395	.37 588	.62 412	.98 807	38
23	.36 449	.37 644	.62 356	.98 804	37
24	.36 502	.37 700	.62 300	.98 801	36
25	.36 555	.37 756	.62 244	.98 798	35
26	.36 608	.37 812	.62 188	.98 795	34
27	.36 660	.37 868	.62 132	.98 792	33
28	.36 713	.37 924	.62 076	.98 789	32
29	.36 766	.37 980	.62 020	.98 786	31
30	.36 819	.38 035	.61 965	.98 783	30
31	.36 871	.38 091	.61 909	.98 780	29
32	.36 924	.38 147	.61 853	.98 777	28
33	.36 976	.38 202	.61 798	.98 774	27
34	.37 028	.38 257	.61 743	.98 771	26
35	.37 081	.38 313	.61 687	.98 768	25
36	.37 133	.38 368	.61 632	.98 765	24
37	.37 185	.38 423	.61 577	.98 762	23
38	.37 237	.38 479	.61 521	.98 759	22
39	.37 289	.38 534	.61 466	.98 756	21
40	.37 341	.38 589	.61 411	.98 753	20
41	.37 393	.38 644	.61 356	.98 750	19
42	.37 445	.38 699	.61 301	.98 746	18
43	.37 497	.38 754	.61 246	.98 743	17
44	.37 549	.38 808	.61 192	.98 740	16
45	.37 600	.38 863	.61 137	.98 737	15
46	.37 652	.38 918	.61 082	.98 734	14
47	.37 703	.38 972	.61 028	.98 731	13
48	.37 755	.39 027	.60 973	.98 728	12
49	.37 806	.39 082	.60 918	.98 725	11
50	.37 858	.39 136	.60 864	.98 722	10
51	.37 909	.39 190	.60 810	.98 719	9
52	.37 960	.39 245	.60 755	.98 715	8
53	.38 011	.39 299	.60 701	.98 712	7
54	.38 062	.39 353	.60 647	.98 709	6
55	.38 113	.39 407	.60 593	.98 706	5
56	.38 164	.39 461	.60 539	.98 703	4
57	.38 215	.39 515	.60 485	.98 700	3
58	.38 266	.39 569	.60 431	.98 697	2
59	.38 317	.39 623	.60 377	.98 694	1
60	.38 368	.39 677	.60 323	.98 690	0
/	°L cos	°L cot	10 L tan	°L sin	/

/	°L sin	°L tan	10 L cot	°L cos	/
0	.38 368	.39 677	.60 323	.98 690	60
1	.38 418	.39 731	.60 269	.98 687	59
2	.38 469	.39 785	.60 215	.98 684	58
3	.38 519	.39 838	.60 162	.98 681	57
4	.38 570	.39 892	.60 108	.98 678	56
5	.38 620	.39 945	.60 055	.98 675	55
6	.38 670	.39 999	.60 001	.98 671	54
7	.38 721	.40 052	.59 948	.98 668	53
8	.38 771	.40 106	.59 894	.98 665	52
9	.38 821	.40 159	.59 841	.98 662	51
10	.38 871	.40 212	.59 788	.98 659	50
11	.38 921	.40 266	.59 734	.98 656	49
12	.38 971	.40 319	.59 681	.98 652	48
13	.39 021	.40 372	.59 628	.98 649	47
14	.39 071	.40 425	.59 575	.98 646	46
15	.39 121	.40 478	.59 522	.98 643	45
16	.39 170	.40 531	.59 469	.98 640	44
17	.39 220	.40 584	.59 416	.98 636	43
18	.39 270	.40 636	.59 364	.98 633	42
19	.39 319	.40 689	.59 311	.98 630	41
20	.39 369	.40 742	.59 258	.98 627	40
21	.39 418	.40 795	.59 205	.98 623	39
22	.39 467	.40 847	.59 153	.98 620	38
23	.39 517	.40 900	.59 100	.98 617	37
24	.39 566	.40 952	.59 048	.98 614	36
25	.39 615	.41 005	.58 995	.98 610	35
26	.39 664	.41 057	.58 943	.98 607	34
27	.39 713	.41 109	.58 891	.98 604	33
28	.39 762	.41 161	.58 839	.98 601	32
29	.39 811	.41 214	.58 786	.98 597	31
30	.39 860	.41 266	.58 734	.98 594	30
31	.39 909	.41 318	.58 682	.98 591	29
32	.39 958	.41 370	.58 630	.98 588	28
33	.40 006	.41 422	.58 578	.98 584	27
34	.40 055	.41 474	.58 526	.98 581	26
35	.40 103	.41 526	.58 474	.98 578	25
36	.40 152	.41 578	.58 422	.98 574	24
37	.40 200	.41 629	.58 371	.98 571	23
38	.40 249	.41 681	.58 319	.98 568	22
39	.40 297	.41 733	.58 267	.98 565	21
40	.40 346	.41 784	.58 216	.98 561	20
41	.40 394	.41 836	.58 164	.98 558	19
42	.40 442	.41 887	.58 113	.98 555	18
43	.40 490	.41 939	.58 061	.98 551	17
44	.40 538	.41 990	.58 010	.98 548	16
45	.40 586	.42 041	.57 959	.98 545	15
46	.40 634	.42 093	.57 907	.98 541	14
47	.40 682	.42 144	.57 856	.98 538	13
48	.40 730	.42 195	.57 805	.98 535	12
49	.40 778	.42 246	.57 754	.98 531	11
50	.40 825	.42 297	.57 703	.98 528	10
51	.40 873	.42 348	.57 652	.98 525	9
52	.40 921	.42 399	.57 601	.98 521	8
53	.40 968	.42 450	.57 550	.98 518	7
54	.41 016	.42 501	.57 499	.98 515	6
55	.41 063	.42 552	.57 448	.98 511	5
56	.41 111	.42 603	.57 397	.98 508	4
57	.41 158	.42 653	.57 347	.98 505	3
58	.41 205	.42 704	.57 296	.98 501	2
59	.41 252	.42 755	.57 245	.98 498	1
60	.41 300	.42 805	.57 195	.98 494	0
/	°L cos	°L cot	10 L tan	°L sin	/

/	θ Lsin	θ Ltan	10Lcot	θ Lcos	/
0	.41300	.42805	.57195	.98494	60
1	.41347	.42856	.57144	.98491	59
2	.41394	.42906	.57094	.98488	58
3	.41441	.42957	.57043	.98484	57
4	.41488	.43007	.56993	.98481	56
5	.41535	.43057	.56943	.98477	55
6	.41582	.43108	.56892	.98474	54
7	.41628	.43158	.56842	.98471	53
8	.41675	.43208	.56792	.98467	52
9	.41722	.43258	.56742	.98464	51
10	.41768	.43308	.56692	.98460	50
11	.41815	.43358	.56642	.98457	49
12	.41861	.43408	.56592	.98453	48
13	.41908	.43458	.56542	.98450	47
14	.41954	.43508	.56492	.98447	46
15	.42001	.43558	.56442	.98443	45
16	.42047	.43607	.56393	.98440	44
17	.42093	.43657	.56343	.98436	43
18	.42140	.43707	.56293	.98433	42
19	.42186	.43756	.56244	.98429	41
20	.42232	.43806	.56194	.98426	40
21	.42278	.43855	.56145	.98422	39
22	.42324	.43905	.56095	.98419	38
23	.42370	.43954	.56046	.98415	37
24	.42416	.44004	.55996	.98412	36
25	.42461	.44053	.55947	.98409	35
26	.42507	.44102	.55898	.98405	34
27	.42553	.44151	.55849	.98402	33
28	.42599	.44201	.55799	.98398	32
29	.42644	.44250	.55750	.98395	31
30	.42690	.44299	.55701	.98391	30
31	.42735	.44348	.55652	.98388	29
32	.42781	.44397	.55603	.98384	28
33	.42826	.44446	.55554	.98381	27
34	.42872	.44495	.55505	.98377	26
35	.42917	.44544	.55456	.98373	25
36	.42962	.44592	.55408	.98370	24
37	.43008	.44641	.55359	.98366	23
38	.43053	.44690	.55310	.98363	22
39	.43098	.44738	.55262	.98359	21
40	.43143	.44787	.55213	.98356	20
41	.43188	.44836	.55164	.98352	19
42	.43233	.44884	.55116	.98349	18
43	.43278	.44933	.55067	.98345	17
44	.43323	.44981	.55019	.98342	16
45	.43367	.45029	.54971	.98338	15
46	.43412	.45078	.54922	.98334	14
47	.43457	.45126	.54874	.98331	13
48	.43502	.45174	.54826	.98327	12
49	.43546	.45222	.54778	.98324	11
50	.43591	.45271	.54729	.98320	10
51	.43635	.45319	.54681	.98317	9
52	.43680	.45367	.54633	.98313	8
53	.43724	.45415	.54585	.98309	7
54	.43769	.45463	.54537	.98306	6
55	.43813	.45511	.54489	.98302	5
56	.43857	.45559	.54441	.98299	4
57	.43901	.45606	.54394	.98295	3
58	.43946	.45654	.54346	.98291	2
59	.43990	.45702	.54298	.98288	1
60	.44034	.45750	.54250	.98284	0
/	θ Lcos	θ Lcot	10Ltan	θ Lsin	/

/	θ Lsin	θ Ltan	10Lcot	θ Lcos	/
0	.44034	.45750	.54250	.98284	60
1	.44078	.45797	.54203	.98281	59
2	.44122	.45845	.54155	.98277	58
3	.44166	.45892	.54108	.98273	57
4	.44210	.45940	.54060	.98270	56
5	.44253	.45987	.54013	.98266	55
6	.44297	.46035	.53965	.98262	54
7	.44341	.46082	.53918	.98259	53
8	.44385	.46130	.53870	.98255	52
9	.44428	.46177	.53823	.98251	51
10	.44472	.46224	.53776	.98248	50
11	.44516	.46271	.53729	.98244	49
12	.44559	.46319	.53681	.98240	48
13	.44602	.46366	.53634	.98237	47
14	.44646	.46413	.53587	.98233	46
15	.44689	.46460	.53540	.98229	45
16	.44733	.46507	.53493	.98226	44
17	.44776	.46554	.53446	.98222	43
18	.44819	.46601	.53399	.98218	42
19	.44862	.46648	.53352	.98215	41
20	.44905	.46694	.53306	.98211	40
21	.44948	.46741	.53259	.98207	39
22	.44992	.46788	.53212	.98204	38
23	.45035	.46835	.53165	.98200	37
24	.45077	.46881	.53119	.98196	36
25	.45120	.46928	.53072	.98192	35
26	.45163	.46975	.53025	.98189	34
27	.45206	.47021	.52979	.98185	33
28	.45249	.47068	.52932	.98181	32
29	.45292	.47114	.52886	.98177	31
30	.45334	.47160	.52840	.98174	30
31	.45377	.47207	.52793	.98170	29
32	.45419	.47253	.52747	.98166	28
33	.45462	.47299	.52701	.98162	27
34	.45504	.47346	.52654	.98159	26
35	.45547	.47392	.52608	.98155	25
36	.45589	.47438	.52562	.98151	24
37	.45632	.47484	.52516	.98147	23
38	.45674	.47530	.52470	.98144	22
39	.45716	.47576	.52424	.98140	21
40	.45758	.47622	.52378	.98136	20
41	.45801	.47668	.52332	.98132	19
42	.45843	.47714	.52286	.98129	18
43	.45885	.47760	.52240	.98125	17
44	.45927	.47806	.52194	.98121	16
45	.45969	.47852	.52148	.98117	15
46	.46011	.47897	.52103	.98113	14
47	.46053	.47943	.52057	.98110	13
48	.46095	.47989	.52011	.98106	12
49	.46136	.48035	.51965	.98102	11
50	.46178	.48080	.51920	.98098	10
51	.46220	.48126	.51874	.98094	9
52	.46262	.48171	.51829	.98090	8
53	.46303	.48217	.51783	.98087	7
54	.46345	.48262	.51738	.98083	6
55	.46386	.48307	.51693	.98079	5
56	.46428	.48353	.51647	.98075	4
57	.46469	.48398	.51602	.98071	3
58	.46511	.48443	.51557	.98067	2
59	.46552	.48489	.51511	.98063	1
60	.46594	.48534	.51466	.98060	0
/	θ Lcos	θ Lcot	10Ltan	θ Lsin	/

/	9Lsin	9Ltan	10Lcot	9Lcos	/
0	.46594	.48534	.51466	.98060	60
1	.46635	.48579	.51421	.98056	59
2	.46676	.48624	.51376	.98052	58
3	.46717	.48669	.51331	.98048	57
4	.46758	.48714	.51286	.98044	56
5	.46800	.48759	.51241	.98040	55
6	.46841	.48804	.51196	.98036	54
7	.46882	.48849	.51151	.98032	53
8	.46923	.48894	.51106	.98029	52
9	.46964	.48939	.51061	.98025	51
10	.47005	.48984	.51016	.98021	50
11	.47045	.49029	.50971	.98017	49
12	.47086	.49073	.50927	.98013	48
13	.47127	.49118	.50882	.98009	47
14	.47168	.49163	.50837	.98005	46
15	.47209	.49207	.50793	.98001	45
16	.47249	.49252	.50748	.97997	44
17	.47290	.49296	.50704	.97993	43
18	.47330	.49341	.50659	.97989	42
19	.47371	.49385	.50615	.97986	41
20	.47411	.49430	.50570	.97982	40
21	.47452	.49474	.50526	.97978	39
22	.47492	.49519	.50481	.97974	38
23	.47533	.49563	.50437	.97970	37
24	.47573	.49607	.50393	.97966	36
25	.47613	.49652	.50348	.97962	35
26	.47654	.49696	.50304	.97958	34
27	.47694	.49740	.50260	.97954	33
28	.47734	.49784	.50216	.97950	32
29	.47774	.49828	.50172	.97946	31
30	.47814	.49872	.50128	.97942	30
31	.47854	.49916	.50084	.97938	29
32	.47894	.49960	.50040	.97934	28
33	.47934	.50004	.49996	.97930	27
34	.47974	.50048	.49952	.97926	26
35	.48014	.50092	.49908	.97922	25
36	.48054	.50136	.49864	.97918	24
37	.48094	.50180	.49820	.97914	23
38	.48133	.50223	.49777	.97910	22
39	.48173	.50267	.49733	.97906	21
40	.48213	.50311	.49689	.97902	20
41	.48252	.50355	.49645	.97898	19
42	.48292	.50398	.49602	.97894	18
43	.48332	.50442	.49558	.97890	17
44	.48371	.50485	.49515	.97886	16
45	.48411	.50529	.49471	.97882	15
46	.48450	.50572	.49428	.97878	14
47	.48490	.50616	.49384	.97874	13
48	.48529	.50659	.49341	.97870	12
49	.48568	.50703	.49297	.97866	11
50	.48607	.50746	.49254	.97861	10
51	.48647	.50789	.49211	.97857	9
52	.48686	.50833	.49167	.97853	8
53	.48725	.50876	.49124	.97849	7
54	.48764	.50919	.49081	.97845	6
55	.48803	.50962	.49038	.97841	5
56	.48842	.51005	.48995	.97837	4
57	.48881	.51048	.48952	.97833	3
58	.48920	.51092	.48908	.97829	2
59	.48959	.51135	.48865	.97825	1
60	.48998	.51178	.48822	.97821	0
/	9Lcos	9Lcot	10Ltan	9Lsin	/

/	9Lsin	9Ltan	10Lcot	9Lcos	/
0	.48998	.51178	.48822	.97821	60
1	.49037	.51221	.48779	.97817	59
2	.49076	.51264	.48736	.97812	58
3	.49115	.51306	.48694	.97808	57
4	.49153	.51349	.48651	.97804	56
5	.49192	.51392	.48608	.97800	55
6	.49231	.51435	.48565	.97796	54
7	.49269	.51478	.48522	.97792	53
8	.49308	.51520	.48480	.97788	52
9	.49347	.51563	.48437	.97784	51
10	.49385	.51606	.48394	.97779	50
11	.49424	.51648	.48352	.97775	49
12	.49462	.51691	.48309	.97771	48
13	.49500	.51734	.48266	.97767	47
14	.49539	.51776	.48224	.97763	46
15	.49577	.51819	.48181	.97759	45
16	.49615	.51861	.48139	.97754	44
17	.49654	.51903	.48097	.97750	43
18	.49692	.51946	.48054	.97746	42
19	.49730	.51988	.48012	.97742	41
20	.49768	.52031	.47969	.97738	40
21	.49806	.52073	.47927	.97734	39
22	.49844	.52115	.47885	.97729	38
23	.49882	.52157	.47843	.97725	37
24	.49920	.52200	.47800	.97721	36
25	.49958	.52242	.47758	.97717	35
26	.49996	.52284	.47716	.97713	34
27	.50034	.52326	.47674	.97708	33
28	.50072	.52368	.47632	.97704	32
29	.50110	.52410	.47590	.97700	31
30	.50148	.52452	.47548	.97696	30
31	.50185	.52494	.47506	.97691	29
32	.50223	.52536	.47464	.97687	28
33	.50261	.52578	.47422	.97683	27
34	.50298	.52620	.47380	.97679	26
35	.50336	.52661	.47339	.97674	25
36	.50374	.52703	.47297	.97670	24
37	.50411	.52745	.47255	.97666	23
38	.50449	.52787	.47213	.97662	22
39	.50486	.52829	.47171	.97657	21
40	.50523	.52870	.47130	.97653	20
41	.50561	.52912	.47088	.97649	19
42	.50598	.52953	.47047	.97645	18
43	.50635	.52995	.47005	.97640	17
44	.50673	.53037	.46963	.97636	16
45	.50710	.53078	.46922	.97632	15
46	.50747	.53120	.46880	.97628	14
47	.50784	.53161	.46839	.97623	13
48	.50821	.53202	.46798	.97619	12
49	.50858	.53244	.46756	.97615	11
50	.50896	.53285	.46715	.97610	10
51	.50933	.53327	.46673	.97606	9
52	.50970	.53368	.46632	.97602	8
53	.51007	.53409	.46591	.97597	7
54	.51043	.53450	.46550	.97593	6
55	.51080	.53492	.46508	.97589	5
56	.51117	.53533	.46467	.97584	4
57	.51154	.53574	.46426	.97580	3
58	.51191	.53615	.46385	.97576	2
59	.51227	.53656	.46344	.97571	1
60	.51264	.53697	.46303	.97567	0
/	9Lcos	9Lcot	10Ltan	9Lsin	/

/	°L sin	°L tan	10 L cot	°L cos	/
0	.51 264	.53 697	.46 303	.97 567	60
1	.51 301	.53 738	.46 262	.97 563	59
2	.51 338	.53 779	.46 221	.97 558	58
3	.51 374	.53 820	.46 180	.97 554	57
4	.51 411	.53 861	.46 139	.97 550	56
5	.51 447	.53 902	.46 098	.97 545	55
6	.51 484	.53 943	.46 057	.97 541	54
7	.51 520	.53 984	.46 016	.97 536	53
8	.51 557	.54 025	.45 975	.97 532	52
9	.51 593	.54 065	.45 935	.97 528	51
10	.51 629	.54 106	.45 894	.97 523	50
11	.51 666	.54 147	.45 853	.97 519	49
12	.51 702	.54 187	.45 813	.97 515	48
13	.51 738	.54 228	.45 772	.97 510	47
14	.51 774	.54 269	.45 731	.97 506	46
15	.51 811	.54 309	.45 691	.97 501	45
16	.51 847	.54 350	.45 650	.97 497	44
17	.51 883	.54 390	.45 610	.97 492	43
18	.51 919	.54 431	.45 569	.97 488	42
19	.51 955	.54 471	.45 529	.97 484	41
20	.51 991	.54 512	.45 488	.97 479	40
21	.52 027	.54 552	.45 448	.97 475	39
22	.52 063	.54 593	.45 407	.97 470	38
23	.52 099	.54 633	.45 367	.97 466	37
24	.52 135	.54 673	.45 327	.97 461	36
25	.52 171	.54 714	.45 286	.97 457	35
26	.52 207	.54 754	.45 246	.97 453	34
27	.52 242	.54 794	.45 206	.97 448	33
28	.52 278	.54 835	.45 165	.97 444	32
29	.52 314	.54 875	.45 125	.97 439	31
30	.52 350	.54 915	.45 085	.97 435	30
31	.52 385	.54 955	.45 045	.97 430	29
32	.52 421	.54 995	.45 005	.97 426	28
33	.52 456	.55 035	.44 965	.97 421	27
34	.52 492	.55 075	.44 925	.97 417	26
35	.52 527	.55 115	.44 885	.97 412	25
36	.52 563	.55 155	.44 845	.97 408	24
37	.52 598	.55 195	.44 805	.97 403	23
38	.52 634	.55 235	.44 765	.97 399	22
39	.52 669	.55 275	.44 725	.97 394	21
40	.52 705	.55 315	.44 685	.97 390	20
41	.52 740	.55 355	.44 645	.97 385	19
42	.52 775	.55 395	.44 605	.97 381	18
43	.52 811	.55 434	.44 566	.97 376	17
44	.52 846	.55 474	.44 526	.97 372	16
45	.52 881	.55 514	.44 486	.97 367	15
46	.52 916	.55 554	.44 446	.97 363	14
47	.52 951	.55 593	.44 407	.97 358	13
48	.52 986	.55 633	.44 367	.97 353	12
49	.53 021	.55 673	.44 327	.97 349	11
50	.53 056	.55 712	.44 288	.97 344	10
51	.53 092	.55 752	.44 248	.97 340	9
52	.53 126	.55 791	.44 209	.97 335	8
53	.53 161	.55 831	.44 169	.97 331	7
54	.53 196	.55 870	.44 130	.97 326	6
55	.53 231	.55 910	.44 090	.97 322	5
56	.53 266	.55 949	.44 051	.97 317	4
57	.53 301	.55 989	.44 011	.97 312	3
58	.53 336	.56 028	.43 972	.97 308	2
59	.53 370	.56 067	.43 933	.97 303	1
60	.53 405	.56 107	.43 893	.97 299	0
/	°L cos	°L cot	10 L tan	°L sin	/

/	°L sin	°L tan	10 L cot	°L cos	/
0	.53 405	.56 107	.43 893	.97 299	60
1	.53 440	.56 146	.43 854	.97 294	59
2	.53 475	.56 185	.43 815	.97 289	58
3	.53 509	.56 224	.43 776	.97 285	57
4	.53 544	.56 264	.43 736	.97 280	56
5	.53 578	.56 303	.43 697	.97 276	55
6	.53 613	.56 342	.43 658	.97 271	54
7	.53 647	.56 381	.43 619	.97 266	53
8	.53 682	.56 420	.43 580	.97 262	52
9	.53 716	.56 459	.43 541	.97 257	51
10	.53 751	.56 498	.43 502	.97 252	50
11	.53 785	.56 537	.43 463	.97 248	49
12	.53 819	.56 576	.43 424	.97 243	48
13	.53 854	.56 615	.43 385	.97 238	47
14	.53 888	.56 654	.43 346	.97 234	46
15	.53 922	.56 693	.43 307	.97 229	45
16	.53 957	.56 732	.43 268	.97 224	44
17	.53 991	.56 771	.43 229	.97 220	43
18	.54 025	.56 810	.43 190	.97 215	42
19	.54 059	.56 849	.43 151	.97 210	41
20	.54 093	.56 887	.43 113	.97 206	40
21	.54 127	.56 926	.43 074	.97 201	39
22	.54 161	.56 965	.43 035	.97 196	38
23	.54 195	.57 004	.42 996	.97 192	37
24	.54 229	.57 042	.42 958	.97 187	36
25	.54 263	.57 081	.42 919	.97 182	35
26	.54 297	.57 120	.42 880	.97 178	34
27	.54 331	.57 158	.42 842	.97 173	33
28	.54 365	.57 197	.42 803	.97 168	32
29	.54 399	.57 235	.42 765	.97 163	31
30	.54 433	.57 274	.42 726	.97 159	30
31	.54 466	.57 312	.42 688	.97 154	29
32	.54 500	.57 351	.42 649	.97 149	28
33	.54 534	.57 389	.42 611	.97 145	27
34	.54 567	.57 428	.42 572	.97 140	26
35	.54 601	.57 466	.42 534	.97 135	25
36	.54 635	.57 504	.42 496	.97 130	24
37	.54 668	.57 543	.42 457	.97 126	23
38	.54 702	.57 581	.42 419	.97 121	22
39	.54 735	.57 619	.42 381	.97 116	21
40	.54 769	.57 658	.42 342	.97 111	20
41	.54 802	.57 696	.42 304	.97 107	19
42	.54 836	.57 734	.42 266	.97 102	18
43	.54 869	.57 772	.42 228	.97 097	17
44	.54 903	.57 810	.42 190	.97 092	16
45	.54 936	.57 849	.42 151	.97 087	15
46	.54 969	.57 887	.42 113	.97 083	14
47	.55 003	.57 925	.42 075	.97 078	13
48	.55 036	.57 963	.42 037	.97 073	12
49	.55 069	.58 001	.41 999	.97 068	11
50	.55 102	.58 039	.41 961	.97 063	10
51	.55 136	.58 077	.41 923	.97 059	9
52	.55 169	.58 115	.41 885	.97 054	8
53	.55 202	.58 153	.41 847	.97 049	7
54	.55 235	.58 191	.41 809	.97 044	6
55	.55 268	.58 229	.41 771	.97 039	5
56	.55 301	.58 267	.41 733	.97 035	4
57	.55 334	.58 304	.41 696	.97 030	3
58	.55 367	.58 342	.41 658	.97 025	2
59	.55 400	.58 380	.41 620	.97 020	1
60	.55 433	.58 418	.41 582	.97 015	0
/	°L cos	°L cot	10 L tan	°L sin	/

/	°L sin	°L tan	10 L cot	°L cos	/
0	.55 433	.58 418	.41 582	.97 015	60
1	.55 466	.58 455	.41 545	.97 010	59
2	.55 499	.58 493	.41 507	.97 005	58
3	.55 532	.58 531	.41 469	.97 001	57
4	.55 564	.58 569	.41 431	.96 996	56
5	.55 597	.58 606	.41 394	.96 991	55
6	.55 630	.58 644	.41 356	.96 986	54
7	.55 663	.58 681	.41 319	.96 981	53
8	.55 695	.58 719	.41 281	.96 976	52
9	.55 728	.58 757	.41 243	.96 971	51
10	.55 761	.58 794	.41 206	.96 966	50
11	.55 793	.58 832	.41 168	.96 962	49
12	.55 826	.58 869	.41 131	.96 957	48
13	.55 858	.58 907	.41 093	.96 952	47
14	.55 891	.58 944	.41 056	.96 947	46
15	.55 923	.58 981	.41 019	.96 942	45
16	.55 956	.59 019	.40 981	.96 937	44
17	.55 988	.59 056	.40 944	.96 932	43
18	.56 021	.59 094	.40 906	.96 927	42
19	.56 053	.59 131	.40 869	.96 922	41
20	.56 085	.59 168	.40 832	.96 917	40
21	.56 118	.59 205	.40 795	.96 912	39
22	.56 150	.59 243	.40 757	.96 907	38
23	.56 182	.59 280	.40 720	.96 903	37
24	.56 215	.59 317	.40 683	.96 898	36
25	.56 247	.59 354	.40 646	.96 893	35
26	.56 279	.59 391	.40 609	.96 888	34
27	.56 311	.59 429	.40 571	.96 883	33
28	.56 343	.59 466	.40 534	.96 878	32
29	.56 375	.59 503	.40 497	.96 873	31
30	.56 408	.59 540	.40 460	.96 868	30
31	.56 440	.59 577	.40 423	.96 863	29
32	.56 472	.59 614	.40 386	.96 858	28
33	.56 504	.59 651	.40 349	.96 853	27
34	.56 536	.59 688	.40 312	.96 848	26
35	.56 568	.59 725	.40 275	.96 843	25
36	.56 599	.59 762	.40 238	.96 838	24
37	.56 631	.59 799	.40 201	.96 833	23
38	.56 663	.59 835	.40 165	.96 828	22
39	.56 695	.59 872	.40 128	.96 823	21
40	.56 727	.59 909	.40 091	.96 818	20
41	.56 759	.59 946	.40 054	.96 813	19
42	.56 790	.59 983	.40 017	.96 808	18
43	.56 822	.60 019	.39 981	.96 803	17
44	.56 854	.60 056	.39 944	.96 798	16
45	.56 886	.60 093	.39 907	.96 793	15
46	.56 917	.60 130	.39 870	.96 788	14
47	.56 949	.60 166	.39 834	.96 783	13
48	.56 980	.60 203	.39 797	.96 778	12
49	.57 012	.60 240	.39 760	.96 772	11
50	.57 044	.60 276	.39 724	.96 767	10
51	.57 075	.60 313	.39 687	.96 762	9
52	.57 107	.60 349	.39 651	.96 757	8
53	.57 138	.60 386	.39 614	.96 752	7
54	.57 169	.60 422	.39 578	.96 747	6
55	.57 201	.60 459	.39 541	.96 742	5
56	.57 232	.60 495	.39 505	.96 737	4
57	.57 264	.60 532	.39 468	.96 732	3
58	.57 295	.60 568	.39 432	.96 727	2
59	.57 326	.60 605	.39 395	.96 722	1
60	.57 358	.60 641	.39 359	.96 717	0
/	°L cos	°L cot	10 L tan	°L sin	/

/	°L sin	°L tan	10 L cot	°L cos	/
0	.57 358	.60 641	.39 359	.96 717	60
1	.57 389	.60 677	.39 323	.96 711	59
2	.57 420	.60 714	.39 286	.96 706	58
3	.57 451	.60 750	.39 250	.96 701	57
4	.57 482	.60 786	.39 214	.96 696	56
5	.57 514	.60 823	.39 177	.96 691	55
6	.57 545	.60 859	.39 141	.96 686	54
7	.57 576	.60 895	.39 105	.96 681	53
8	.57 607	.60 931	.39 069	.96 676	52
9	.57 638	.60 967	.39 033	.96 670	51
10	.57 669	.61 004	.38 996	.96 665	50
11	.57 700	.61 040	.38 960	.96 660	49
12	.57 731	.61 076	.38 924	.96 655	48
13	.57 762	.61 112	.38 888	.96 650	47
14	.57 793	.61 148	.38 852	.96 645	46
15	.57 824	.61 184	.38 816	.96 640	45
16	.57 855	.61 220	.38 780	.96 634	44
17	.57 885	.61 256	.38 744	.96 629	43
18	.57 916	.61 292	.38 708	.96 624	42
19	.57 947	.61 328	.38 672	.96 619	41
20	.57 978	.61 364	.38 636	.96 614	40
21	.58 008	.61 400	.38 600	.96 608	39
22	.58 039	.61 436	.38 564	.96 603	38
23	.58 070	.61 472	.38 528	.96 598	37
24	.58 101	.61 508	.38 492	.96 593	36
25	.58 131	.61 544	.38 456	.96 588	35
26	.58 162	.61 579	.38 421	.96 582	34
27	.58 192	.61 615	.38 385	.96 577	33
28	.58 223	.61 651	.38 349	.96 572	32
29	.58 253	.61 687	.38 313	.96 567	31
30	.58 284	.61 722	.38 278	.96 562	30
31	.58 314	.61 758	.38 242	.96 556	29
32	.58 345	.61 794	.38 206	.96 551	28
33	.58 375	.61 830	.38 170	.96 546	27
34	.58 406	.61 865	.38 135	.96 541	26
35	.58 436	.61 901	.38 099	.96 535	25
36	.58 467	.61 936	.38 064	.96 530	24
37	.58 497	.61 972	.38 028	.96 525	23
38	.58 527	.62 008	.37 992	.96 520	22
39	.58 557	.62 043	.37 957	.96 514	21
40	.58 588	.62 079	.37 921	.96 509	20
41	.58 618	.62 114	.37 886	.96 504	19
42	.58 648	.62 150	.37 850	.96 498	18
43	.58 678	.62 185	.37 815	.96 493	17
44	.58 709	.62 221	.37 779	.96 488	16
45	.58 739	.62 256	.37 744	.96 483	15
46	.58 769	.62 292	.37 708	.96 477	14
47	.58 799	.62 327	.37 673	.96 472	13
48	.58 829	.62 362	.37 638	.96 467	12
49	.58 859	.62 398	.37 602	.96 461	11
50	.58 889	.62 433	.37 567	.96 456	10
51	.58 919	.62 468	.37 532	.96 451	9
52	.58 949	.62 504	.37 496	.96 445	8
53	.58 979	.62 539	.37 461	.96 440	7
54	.59 009	.62 574	.37 426	.96 435	6
55	.59 039	.62 609	.37 391	.96 429	5
56	.59 069	.62 645	.37 355	.96 424	4
57	.59 098	.62 680	.37 320	.96 419	3
58	.59 128	.62 715	.37 285	.96 413	2
59	.59 158	.62 750	.37 250	.96 408	1
60	.59 188	.62 785	.37 215	.96 403	0
/	°L cos	°L cot	10 L tan	°L sin	/

/	°L sin	°L tan	10 Lcot	°L cos	/
0	.59188	.62785	.37215	.96403	60
1	.59218	.62820	.37180	.96397	59
2	.59247	.62855	.37145	.96392	58
3	.59277	.62890	.37110	.96387	57
4	.59307	.62926	.37074	.96381	56
5	.59336	.62961	.37039	.96376	55
6	.59366	.62996	.37004	.96370	54
7	.59396	.63031	.36969	.96365	53
8	.59425	.63066	.36934	.96360	52
9	.59455	.63101	.36899	.96354	51
10	.59484	.63135	.36865	.96349	50
11	.59514	.63170	.36830	.96343	49
12	.59543	.63205	.36795	.96338	48
13	.59573	.63240	.36760	.96333	47
14	.59602	.63275	.36725	.96327	46
15	.59632	.63310	.36690	.96322	45
16	.59661	.63345	.36655	.96316	44
17	.59690	.63379	.36621	.96311	43
18	.59720	.63414	.36586	.96305	42
19	.59749	.63449	.36551	.96300	41
20	.59778	.63484	.36516	.96294	40
21	.59808	.63519	.36481	.96289	39
22	.59837	.63553	.36447	.96284	38
23	.59866	.63588	.36412	.96278	37
24	.59895	.63623	.36377	.96273	36
25	.59924	.63657	.36343	.96267	35
26	.59954	.63692	.36308	.96262	34
27	.59983	.63726	.36274	.96256	33
28	.60012	.63761	.36239	.96251	32
29	.60041	.63796	.36204	.96245	31
30	.60070	.63830	.36170	.96240	30
31	.60099	.63865	.36135	.96234	29
32	.60128	.63899	.36101	.96229	28
33	.60157	.63934	.36066	.96223	27
34	.60186	.63968	.36032	.96218	26
35	.60215	.64003	.35997	.96212	25
36	.60244	.64037	.35963	.96207	24
37	.60273	.64072	.35928	.96201	23
38	.60302	.64106	.35894	.96196	22
39	.60331	.64140	.35860	.96190	21
40	.60359	.64175	.35825	.96185	20
41	.60388	.64209	.35791	.96179	19
42	.60417	.64243	.35757	.96174	18
43	.60446	.64278	.35722	.96168	17
44	.60474	.64312	.35688	.96162	16
45	.60503	.64346	.35654	.96157	15
46	.60532	.64381	.35619	.96151	14
47	.60561	.64415	.35585	.96146	13
48	.60589	.64449	.35551	.96140	12
49	.60618	.64483	.35517	.96135	11
50	.60646	.64517	.35483	.96129	10
51	.60675	.64552	.35448	.96123	9
52	.60704	.64586	.35414	.96118	8
53	.60732	.64620	.35380	.96112	7
54	.60761	.64654	.35346	.96107	6
55	.60789	.64688	.35312	.96101	5
56	.60818	.64722	.35278	.96095	4
57	.60846	.64756	.35244	.96090	3
58	.60875	.64790	.35210	.96084	2
59	.60903	.64824	.35176	.96079	1
60	.60931	.64858	.35142	.96073	0
/	°L cos	°L cot	10 Ltan	°L sin	/

/	°L sin	°L tan	10 Lcot	°L cos	/
0	.60931	.64858	.35142	.96073	60
1	.60960	.64892	.35108	.96067	59
2	.60988	.64926	.35074	.96062	58
3	.61016	.64960	.35040	.96056	57
4	.61045	.64994	.35006	.96050	56
5	.61073	.65028	.34972	.96045	55
6	.61101	.65062	.34938	.96039	54
7	.61129	.65096	.34904	.96034	53
8	.61158	.65130	.34870	.96028	52
9	.61186	.65164	.34836	.96022	51
10	.61214	.65197	.34803	.96017	50
11	.61242	.65231	.34769	.96011	49
12	.61270	.65265	.34735	.96005	48
13	.61298	.65299	.34701	.96000	47
14	.61326	.65333	.34667	.95994	46
15	.61354	.65366	.34634	.95988	45
16	.61382	.65400	.34600	.95982	44
17	.61411	.65434	.34566	.95977	43
18	.61438	.65467	.34533	.95971	42
19	.61466	.65501	.34499	.95965	41
20	.61494	.65535	.34465	.95960	40
21	.61522	.65568	.34432	.95954	39
22	.61550	.65602	.34398	.95948	38
23	.61578	.65636	.34364	.95942	37
24	.61606	.65669	.34331	.95937	36
25	.61634	.65703	.34297	.95931	35
26	.61662	.65736	.34264	.95925	34
27	.61689	.65770	.34230	.95920	33
28	.61717	.65803	.34197	.95914	32
29	.61745	.65837	.34163	.95908	31
30	.61773	.65870	.34130	.95902	30
31	.61800	.65904	.34096	.95897	29
32	.61828	.65937	.34063	.95891	28
33	.61856	.65971	.34029	.95885	27
34	.61883	.66004	.33996	.95879	26
35	.61911	.66038	.33962	.95873	25
36	.61939	.66071	.33929	.95868	24
37	.61966	.66104	.33896	.95862	23
38	.61994	.66138	.33862	.95856	22
39	.62021	.66171	.33829	.95850	21
40	.62049	.66204	.33796	.95844	20
41	.62076	.66238	.33762	.95839	19
42	.62104	.66271	.33729	.95833	18
43	.62131	.66304	.33696	.95827	17
44	.62159	.66337	.33663	.95821	16
45	.62186	.66371	.33629	.95815	15
46	.62214	.66404	.33596	.95810	14
47	.62241	.66437	.33563	.95804	13
48	.62268	.66470	.33530	.95798	12
49	.62296	.66503	.33497	.95792	11
50	.62323	.66537	.33463	.95786	10
51	.62350	.66570	.33430	.95780	9
52	.62377	.66603	.33397	.95775	8
53	.62405	.66636	.33364	.95769	7
54	.62432	.66669	.33331	.95763	6
55	.62459	.66702	.33298	.95757	5
56	.62486	.66735	.33265	.95751	4
57	.62513	.66768	.33232	.95745	3
58	.62541	.66801	.33199	.95739	2
59	.62568	.66834	.33166	.95733	1
60	.62595	.66867	.33133	.95728	0
/	°L cos	°L cot	10 Ltan	°L sin	/

25°

xxxiii

/	9L sin	9L tan	10L cot	9L cos	/
0	.62595	.66867	.33133	.95728	60
1	.62622	.66900	.33100	.95722	59
2	.62649	.66933	.33067	.95716	58
3	.62676	.66966	.33034	.95710	57
4	.62703	.66999	.33001	.95704	56
5	.62730	.67032	.32968	.95698	55
6	.62757	.67065	.32935	.95692	54
7	.62784	.67098	.32902	.95686	53
8	.62811	.67131	.32869	.95680	52
9	.62838	.67163	.32837	.95674	51
10	.62865	.67196	.32804	.95668	50
11	.62892	.67229	.32771	.95663	49
12	.62918	.67262	.32738	.95657	48
13	.62945	.67295	.32705	.95651	47
14	.62972	.67327	.32673	.95645	46
15	.62999	.67360	.32640	.95639	45
16	.63026	.67393	.32607	.95633	44
17	.63052	.67426	.32574	.95627	43
18	.63079	.67458	.32542	.95621	42
19	.63106	.67491	.32509	.95615	41
20	.63133	.67524	.32476	.95609	40
21	.63159	.67556	.32444	.95603	39
22	.63186	.67589	.32411	.95597	38
23	.63213	.67622	.32378	.95591	37
24	.63239	.67654	.32346	.95585	36
25	.63266	.67687	.32313	.95579	35
26	.63292	.67719	.32281	.95573	34
27	.63319	.67752	.32248	.95567	33
28	.63345	.67785	.32215	.95561	32
29	.63372	.67817	.32183	.95555	31
30	.63398	.67850	.32150	.95549	30
31	.63425	.67882	.32118	.95543	29
32	.63451	.67915	.32085	.95537	28
33	.63478	.67947	.32053	.95531	27
34	.63504	.67980	.32020	.95525	26
35	.63531	.68012	.31988	.95519	25
36	.63557	.68044	.31956	.95513	24
37	.63583	.68077	.31923	.95507	23
38	.63610	.68109	.31891	.95500	22
39	.63636	.68142	.31858	.95494	21
40	.63662	.68174	.31826	.95488	20
41	.63689	.68206	.31794	.95482	19
42	.63715	.68239	.31761	.95476	18
43	.63741	.68271	.31729	.95470	17
44	.63767	.68303	.31697	.95464	16
45	.63794	.68336	.31664	.95458	15
46	.63820	.68368	.31632	.95452	14
47	.63846	.68400	.31600	.95446	13
48	.63872	.68432	.31568	.95440	12
49	.63898	.68465	.31535	.95434	11
50	.63924	.68497	.31503	.95427	10
51	.63950	.68529	.31471	.95421	9
52	.63976	.68561	.31439	.95415	8
53	.64002	.68593	.31407	.95409	7
54	.64028	.68626	.31374	.95403	6
55	.64054	.68658	.31342	.95397	5
56	.64080	.68690	.31310	.95391	4
57	.64106	.68722	.31278	.95384	3
58	.64132	.68754	.31246	.95378	2
59	.64158	.68786	.31214	.95372	1
60	.64184	.68818	.31182	.95366	0
/	9L cos	9L cot	10L tan	9L sin	/

64°

26°

/	9L sin	9L tan	10L cot	9L cos	/
0	.64184	.68818	.31182	.95366	60
1	.64210	.68850	.31150	.95360	59
2	.64236	.68882	.31118	.95354	58
3	.64262	.68914	.31086	.95348	57
4	.64288	.68946	.31054	.95341	56
5	.64313	.68978	.31022	.95335	55
6	.64339	.69010	.30990	.95329	54
7	.64365	.69042	.30958	.95323	53
8	.64391	.69074	.30926	.95317	52
9	.64417	.69106	.30894	.95310	51
10	.64442	.69138	.30862	.95304	50
11	.64468	.69170	.30830	.95298	49
12	.64494	.69202	.30798	.95292	48
13	.64519	.69234	.30766	.95286	47
14	.64545	.69266	.30734	.95279	46
15	.64571	.69298	.30702	.95273	45
16	.64596	.69329	.30671	.95267	44
17	.64622	.69361	.30639	.95261	43
18	.64647	.69393	.30607	.95254	42
19	.64673	.69425	.30575	.95248	41
20	.64698	.69457	.30543	.95242	40
21	.64724	.69488	.30512	.95236	39
22	.64749	.69520	.30480	.95230	38
23	.64775	.69552	.30448	.95223	37
24	.64800	.69584	.30416	.95217	36
25	.64826	.69615	.30385	.95211	35
26	.64851	.69647	.30353	.95204	34
27	.64877	.69679	.30321	.95198	33
28	.64902	.69710	.30290	.95192	32
29	.64927	.69742	.30258	.95185	31
30	.64953	.69774	.30226	.95179	30
31	.64978	.69805	.30195	.95173	29
32	.65003	.69837	.30163	.95167	28
33	.65029	.69868	.30132	.95160	27
34	.65054	.69900	.30100	.95154	26
35	.65079	.69932	.30068	.95148	25
36	.65104	.69963	.30037	.95141	24
37	.65130	.69995	.30005	.95135	23
38	.65155	.70026	.29974	.95129	22
39	.65180	.70058	.29942	.95122	21
40	.65205	.70089	.29911	.95116	20
41	.65230	.70121	.29879	.95110	19
42	.65255	.70152	.29848	.95103	18
43	.65281	.70184	.29816	.95097	17
44	.65306	.70215	.29785	.95090	16
45	.65331	.70247	.29753	.95084	15
46	.65356	.70278	.29722	.95078	14
47	.65381	.70309	.29691	.95071	13
48	.65406	.70341	.29659	.95065	12
49	.65431	.70372	.29628	.95059	11
50	.65456	.70404	.29596	.95052	10
51	.65481	.70435	.29565	.95046	9
52	.65506	.70466	.29534	.95039	8
53	.65531	.70498	.29502	.95033	7
54	.65556	.70529	.29471	.95027	6
55	.65580	.70560	.29440	.95020	5
56	.65605	.70592	.29408	.95014	4
57	.65630	.70623	.29377	.95007	3
58	.65655	.70654	.29346	.95001	2
59	.65680	.70685	.29315	.94995	1
60	.65705	.70717	.29283	.94988	0
/	9L cos	9L cot	10L tan	9L sin	/

63°

/	θ Lsin	θ Ltan	10Lcot	θ Lcos	/
0	.65705	.70717	.29283	.94988	60
1	.65729	.70748	.29252	.94982	59
2	.65754	.70779	.29221	.94975	58
3	.65779	.70810	.29190	.94969	57
4	.65804	.70841	.29159	.94962	56
5	.65828	.70873	.29127	.94956	55
6	.65853	.70904	.29096	.94949	54
7	.65878	.70935	.29065	.94943	53
8	.65902	.70966	.29034	.94936	52
9	.65927	.70997	.29003	.94930	51
10	.65952	.71028	.28972	.94923	50
11	.65976	.71059	.28941	.94917	49
12	.66001	.71090	.28910	.94911	48
13	.66025	.71121	.28879	.94904	47
14	.66050	.71153	.28847	.94898	46
15	.66075	.71184	.28816	.94891	45
16	.66099	.71215	.28785	.94885	44
17	.66124	.71246	.28754	.94878	43
18	.66148	.71277	.28723	.94871	42
19	.66173	.71308	.28692	.94865	41
20	.66197	.71339	.28661	.94858	40
21	.66221	.71370	.28630	.94852	39
22	.66246	.71401	.28599	.94845	38
23	.66270	.71431	.28569	.94839	37
24	.66295	.71462	.28538	.94832	36
25	.66319	.71493	.28507	.94826	35
26	.66343	.71524	.28476	.94819	34
27	.66368	.71555	.28445	.94813	33
28	.66392	.71586	.28414	.94806	32
29	.66416	.71617	.28383	.94799	31
30	.66441	.71648	.28352	.94793	30
31	.66465	.71679	.28321	.94786	29
32	.66489	.71709	.28291	.94780	28
33	.66513	.71740	.28260	.94773	27
34	.66537	.71771	.28229	.94767	26
35	.66562	.71802	.28198	.94760	25
36	.66586	.71833	.28167	.94753	24
37	.66610	.71863	.28137	.94747	23
38	.66634	.71894	.28106	.94740	22
39	.66658	.71925	.28075	.94734	21
40	.66682	.71955	.28045	.94727	20
41	.66706	.71986	.28014	.94720	19
42	.66731	.72017	.27983	.94714	18
43	.66755	.72048	.27952	.94707	17
44	.66779	.72078	.27922	.94700	16
45	.66803	.72109	.27891	.94694	15
46	.66827	.72140	.27860	.94687	14
47	.66851	.72170	.27830	.94680	13
48	.66875	.72201	.27799	.94674	12
49	.66899	.72231	.27769	.94667	11
50	.66922	.72262	.27738	.94660	10
51	.66946	.72293	.27707	.94654	9
52	.66970	.72323	.27677	.94647	8
53	.66994	.72354	.27646	.94640	7
54	.67018	.72384	.27616	.94634	6
55	.67042	.72415	.27585	.94627	5
56	.67066	.72445	.27555	.94620	4
57	.67090	.72476	.27524	.94614	3
58	.67113	.72506	.27494	.94607	2
59	.67137	.72537	.27463	.94600	1
60	.67161	.72567	.27433	.94593	0
/	θ Lcos	θ Lcot	10Ltan	θ Lsin	/

/	θ Lsin	θ Ltan	10Lcot	θ Lcos	/
0	.67161	.72567	.27433	.94593	60
1	.67185	.72598	.27402	.94587	59
2	.67208	.72628	.27372	.94580	58
3	.67232	.72659	.27341	.94573	57
4	.67256	.72689	.27311	.94567	56
5	.67280	.72720	.27280	.94560	55
6	.67303	.72750	.27250	.94553	54
7	.67327	.72780	.27220	.94546	53
8	.67350	.72811	.27189	.94540	52
9	.67374	.72841	.27159	.94533	51
10	.67398	.72872	.27128	.94526	50
11	.67421	.72902	.27098	.94519	49
12	.67445	.72932	.27068	.94513	48
13	.67468	.72963	.27037	.94506	47
14	.67492	.72993	.27007	.94499	46
15	.67515	.73023	.26977	.94492	45
16	.67539	.73054	.26946	.94485	44
17	.67562	.73084	.26916	.94479	43
18	.67586	.73114	.26886	.94472	42
19	.67609	.73144	.26856	.94465	41
20	.67633	.73175	.26825	.94458	40
21	.67656	.73205	.26795	.94451	39
22	.67680	.73235	.26765	.94445	38
23	.67703	.73265	.26735	.94438	37
24	.67726	.73295	.26705	.94431	36
25	.67750	.73326	.26674	.94424	35
26	.67773	.73356	.26644	.94417	34
27	.67796	.73386	.26614	.94410	33
28	.67820	.73416	.26584	.94404	32
29	.67843	.73446	.26554	.94397	31
30	.67866	.73476	.26524	.94390	30
31	.67890	.73507	.26493	.94383	29
32	.67913	.73537	.26463	.94376	28
33	.67936	.73567	.26433	.94369	27
34	.67959	.73597	.26403	.94362	26
35	.67982	.73627	.26373	.94355	25
36	.68006	.73657	.26343	.94349	24
37	.68029	.73687	.26313	.94342	23
38	.68052	.73717	.26283	.94335	22
39	.68075	.73747	.26253	.94328	21
40	.68098	.73777	.26223	.94321	20
41	.68121	.73807	.26193	.94314	19
42	.68144	.73837	.26163	.94307	18
43	.68167	.73867	.26133	.94300	17
44	.68190	.73897	.26103	.94293	16
45	.68213	.73927	.26073	.94286	15
46	.68237	.73957	.26043	.94279	14
47	.68260	.73987	.26013	.94273	13
48	.68283	.74017	.25983	.94266	12
49	.68305	.74047	.25953	.94259	11
50	.68328	.74077	.25923	.94252	10
51	.68351	.74107	.25893	.94245	9
52	.68374	.74137	.25863	.94238	8
53	.68397	.74166	.25834	.94231	7
54	.68420	.74196	.25804	.94224	6
55	.68443	.74226	.25774	.94217	5
56	.68466	.74256	.25744	.94210	4
57	.68489	.74286	.25714	.94203	3
58	.68512	.74316	.25684	.94196	2
59	.68534	.74345	.25655	.94189	1
60	.68557	.74375	.25625	.94182	0
/	θ Lcos	θ Lcot	10Ltan	θ Lsin	/

29°

30°

XXXV

/	9 L sin	9 L tan	10 L cot	9 L cos	/
0	.68557	.74375	.25625	.94182	60
1	.68580	.74405	.25595	.94175	59
2	.68603	.74435	.25565	.94168	58
3	.68625	.74465	.25535	.94161	57
4	.68648	.74494	.25506	.94154	56
5	.68671	.74524	.25476	.94147	55
6	.68694	.74554	.25446	.94140	54
7	.68716	.74583	.25417	.94133	53
8	.68739	.74613	.25387	.94126	52
9	.68762	.74643	.25357	.94119	51
10	.68784	.74673	.25327	.94112	50
11	.68807	.74702	.25298	.94105	49
12	.68829	.74732	.25268	.94098	48
13	.68852	.74762	.25238	.94090	47
14	.68875	.74791	.25209	.94083	46
15	.68897	.74821	.25179	.94076	45
16	.68920	.74851	.25149	.94069	44
17	.68942	.74880	.25120	.94062	43
18	.68965	.74910	.25090	.94055	42
19	.68987	.74939	.25061	.94048	41
20	.69010	.74969	.25031	.94041	40
21	.69032	.74998	.25002	.94034	39
22	.69055	.75028	.24972	.94027	38
23	.69077	.75058	.24942	.94020	37
24	.69100	.75087	.24913	.94012	36
25	.69122	.75117	.24883	.94005	35
26	.69144	.75146	.24854	.93998	34
27	.69167	.75176	.24824	.93991	33
28	.69189	.75205	.24795	.93984	32
29	.69212	.75235	.24765	.93977	31
30	.69234	.75264	.24736	.93970	30
31	.69256	.75294	.24706	.93963	29
32	.69279	.75323	.24677	.93955	28
33	.69301	.75353	.24647	.93948	27
34	.69323	.75382	.24618	.93941	26
35	.69345	.75411	.24589	.93934	25
36	.69368	.75441	.24559	.93927	24
37	.69390	.75470	.24530	.93920	23
38	.69412	.75500	.24500	.93912	22
39	.69434	.75529	.24471	.93905	21
40	.69456	.75558	.24442	.93898	20
41	.69479	.75588	.24412	.93891	19
42	.69501	.75617	.24383	.93884	18
43	.69523	.75647	.24353	.93876	17
44	.69545	.75676	.24324	.93869	16
45	.69567	.75705	.24295	.93862	15
46	.69589	.75735	.24265	.93855	14
47	.69611	.75764	.24236	.93847	13
48	.69633	.75793	.24207	.93840	12
49	.69655	.75822	.24178	.93833	11
50	.69677	.75852	.24148	.93826	10
51	.69699	.75881	.24119	.93819	9
52	.69721	.75910	.24090	.93811	8
53	.69743	.75939	.24061	.93804	7
54	.69765	.75969	.24031	.93797	6
55	.69787	.75998	.24002	.93789	5
56	.69809	.76027	.23973	.93782	4
57	.69831	.76056	.23944	.93775	3
58	.69853	.76086	.23914	.93768	2
59	.69875	.76115	.23885	.93760	1
60	.69897	.76144	.23856	.93753	0
/	9 L cos	9 L cot	10 L tan	9 L sin	/

60°

/	9 L sin	9 L tan	10 L cot	9 L cos	/
0	.69897	.76144	.23856	.93753	60
1	.69919	.76173	.23827	.93746	59
2	.69941	.76202	.23798	.93738	58
3	.69963	.76231	.23769	.93731	57
4	.69984	.76261	.23739	.93724	56
5	.70006	.76290	.23710	.93717	55
6	.70028	.76319	.23681	.93709	54
7	.70050	.76348	.23652	.93702	53
8	.70072	.76377	.23623	.93695	52
9	.70093	.76406	.23594	.93687	51
10	.70115	.76435	.23565	.93680	50
11	.70137	.76464	.23536	.93673	49
12	.70159	.76493	.23507	.93665	48
13	.70180	.76522	.23478	.93658	47
14	.70202	.76551	.23449	.93650	46
15	.70224	.76580	.23420	.93643	45
16	.70245	.76609	.23391	.93636	44
17	.70267	.76639	.23361	.93628	43
18	.70288	.76668	.23332	.93621	42
19	.70310	.76697	.23303	.93614	41
20	.70332	.76725	.23275	.93606	40
21	.70353	.76754	.23246	.93599	39
22	.70375	.76783	.23217	.93591	38
23	.70396	.76812	.23188	.93584	37
24	.70418	.76841	.23159	.93577	36
25	.70439	.76870	.23130	.93569	35
26	.70461	.76899	.23101	.93562	34
27	.70482	.76928	.23072	.93554	33
28	.70504	.76957	.23043	.93547	32
29	.70525	.76986	.23014	.93539	31
30	.70547	.77015	.22985	.93532	30
31	.70568	.77044	.22956	.93525	29
32	.70590	.77073	.22927	.93517	28
33	.70611	.77101	.22899	.93510	27
34	.70633	.77130	.22870	.93502	26
35	.70654	.77159	.22841	.93495	25
36	.70675	.77188	.22812	.93487	24
37	.70697	.77217	.22783	.93480	23
38	.70718	.77246	.22754	.93472	22
39	.70739	.77274	.22726	.93465	21
40	.70761	.77303	.22697	.93457	20
41	.70782	.77332	.22668	.93450	19
42	.70803	.77361	.22639	.93442	18
43	.70824	.77390	.22610	.93435	17
44	.70846	.77418	.22582	.93427	16
45	.70867	.77447	.22553	.93420	15
46	.70888	.77476	.22524	.93412	14
47	.70909	.77505	.22495	.93405	13
48	.70931	.77533	.22467	.93397	12
49	.70952	.77562	.22438	.93390	11
50	.70973	.77591	.22409	.93382	10
51	.70994	.77619	.22381	.93375	9
52	.71015	.77648	.22352	.93367	8
53	.71036	.77677	.22323	.93360	7
54	.71058	.77706	.22294	.93352	6
55	.71079	.77734	.22266	.93344	5
56	.71100	.77763	.22237	.93337	4
57	.71121	.77791	.22209	.93329	3
58	.71142	.77820	.22180	.93322	2
59	.71163	.77849	.22151	.93314	1
60	.71184	.77877	.22123	.93307	0
/	9 L cos	9 L cot	10 L tan	9 L sin	/

59°

/	θ L sin	θ L tan	10 Lcot	θ Lcos	/
0	.71 184	.77 877	.22 123	.93 307	60
1	.71 205	.77 906	.22 094	.93 299	59
2	.71 226	.77 935	.22 065	.93 291	58
3	.71 247	.77 963	.22 037	.93 284	57
4	.71 268	.77 992	.22 008	.93 276	56
5	.71 289	.78 020	.21 980	.93 269	55
6	.71 310	.78 049	.21 951	.93 261	54
7	.71 331	.78 077	.21 923	.93 253	53
8	.71 352	.78 106	.21 894	.93 246	52
9	.71 373	.78 135	.21 865	.93 238	51
10	.71 393	.78 163	.21 837	.93 230	50
11	.71 414	.78 192	.21 808	.93 223	49
12	.71 435	.78 220	.21 780	.93 215	48
13	.71 456	.78 249	.21 751	.93 207	47
14	.71 477	.78 277	.21 723	.93 200	46
15	.71 498	.78 306	.21 694	.93 192	45
16	.71 519	.78 334	.21 666	.93 184	44
17	.71 539	.78 363	.21 637	.93 177	43
18	.71 560	.78 391	.21 609	.93 169	42
19	.71 581	.78 419	.21 581	.93 161	41
20	.71 602	.78 448	.21 552	.93 154	40
21	.71 622	.78 476	.21 524	.93 146	39
22	.71 643	.78 505	.21 495	.93 138	38
23	.71 664	.78 533	.21 467	.93 131	37
24	.71 685	.78 562	.21 438	.93 123	36
25	.71 705	.78 590	.21 410	.93 115	35
26	.71 726	.78 618	.21 382	.93 108	34
27	.71 747	.78 647	.21 353	.93 100	33
28	.71 767	.78 675	.21 325	.93 092	32
29	.71 788	.78 704	.21 296	.93 084	31
30	.71 809	.78 732	.21 268	.93 077	30
31	.71 829	.78 760	.21 240	.93 069	29
32	.71 850	.78 789	.21 211	.93 061	28
33	.71 870	.78 817	.21 183	.93 053	27
34	.71 891	.78 845	.21 155	.93 046	26
35	.71 911	.78 874	.21 126	.93 038	25
36	.71 932	.78 902	.21 098	.93 030	24
37	.71 952	.78 930	.21 070	.93 022	23
38	.71 973	.78 959	.21 041	.93 014	22
39	.71 994	.78 987	.21 013	.93 007	21
40	.72 014	.79 015	.20 985	.92 999	20
41	.72 034	.79 043	.20 957	.92 991	19
42	.72 055	.79 072	.20 928	.92 983	18
43	.72 075	.79 100	.20 900	.92 976	17
44	.72 096	.79 128	.20 872	.92 968	16
45	.72 116	.79 156	.20 844	.92 960	15
46	.72 137	.79 185	.20 815	.92 952	14
47	.72 157	.79 213	.20 787	.92 944	13
48	.72 177	.79 241	.20 759	.92 936	12
49	.72 198	.79 269	.20 731	.92 929	11
50	.72 218	.79 297	.20 703	.92 921	10
51	.72 238	.79 326	.20 674	.92 913	9
52	.72 259	.79 354	.20 646	.92 905	8
53	.72 279	.79 382	.20 618	.92 897	7
54	.72 299	.79 410	.20 590	.92 889	6
55	.72 320	.79 438	.20 562	.92 881	5
56	.72 340	.79 466	.20 534	.92 874	4
57	.72 360	.79 495	.20 505	.92 866	3
58	.72 381	.79 523	.20 477	.92 858	2
59	.72 401	.79 551	.20 449	.92 850	1
60	.72 421	.79 579	.20 421	.92 842	0
/	θ Lcos	θ Lcot	10 Ltan	θ Lsin	/

/	θ L sin	θ L tan	10 Lcot	θ Lcos	/
0	.72 421	.79 579	.20 421	.92 842	60
1	.72 441	.79 607	.20 393	.92 834	59
2	.72 461	.79 635	.20 365	.92 826	58
3	.72 482	.79 663	.20 337	.92 818	57
4	.72 502	.79 691	.20 309	.92 810	56
5	.72 522	.79 719	.20 281	.92 803	55
6	.72 542	.79 747	.20 253	.92 795	54
7	.72 562	.79 776	.20 224	.92 787	53
8	.72 582	.79 804	.20 196	.92 779	52
9	.72 602	.79 832	.20 168	.92 771	51
10	.72 622	.79 860	.20 140	.92 763	50
11	.72 643	.79 888	.20 112	.92 755	49
12	.72 663	.79 916	.20 084	.92 747	48
13	.72 683	.79 944	.20 056	.92 739	47
14	.72 703	.79 972	.20 028	.92 731	46
15	.72 723	.80 000	.20 000	.92 723	45
16	.72 743	.80 028	.19 972	.92 715	44
17	.72 763	.80 056	.19 944	.92 707	43
18	.72 783	.80 084	.19 916	.92 699	42
19	.72 803	.80 112	.19 888	.92 691	41
20	.72 823	.80 140	.19 860	.92 683	40
21	.72 843	.80 168	.19 832	.92 675	39
22	.72 863	.80 195	.19 805	.92 667	38
23	.72 883	.80 223	.19 777	.92 659	37
24	.72 902	.80 251	.19 749	.92 651	36
25	.72 922	.80 279	.19 721	.92 643	35
26	.72 942	.80 307	.19 693	.92 635	34
27	.72 962	.80 335	.19 665	.92 627	33
28	.72 982	.80 363	.19 637	.92 619	32
29	.73 002	.80 391	.19 609	.92 611	31
30	.73 022	.80 419	.19 581	.92 603	30
31	.73 041	.80 447	.19 553	.92 595	29
32	.73 061	.80 474	.19 526	.92 587	28
33	.73 081	.80 502	.19 498	.92 579	27
34	.73 101	.80 530	.19 470	.92 571	26
35	.73 121	.80 558	.19 442	.92 563	25
36	.73 140	.80 586	.19 414	.92 555	24
37	.73 160	.80 614	.19 386	.92 546	23
38	.73 180	.80 642	.19 358	.92 538	22
39	.73 200	.80 669	.19 331	.92 530	21
40	.73 219	.80 697	.19 303	.92 522	20
41	.73 239	.80 725	.19 275	.92 514	19
42	.73 259	.80 753	.19 247	.92 506	18
43	.73 278	.80 781	.19 219	.92 498	17
44	.73 298	.80 808	.19 192	.92 490	16
45	.73 318	.80 836	.19 164	.92 482	15
46	.73 337	.80 864	.19 136	.92 473	14
47	.73 357	.80 892	.19 108	.92 465	13
48	.73 377	.80 919	.19 081	.92 457	12
49	.73 396	.80 947	.19 053	.92 449	11
50	.73 416	.80 975	.19 025	.92 441	10
51	.73 435	.81 003	.18 997	.92 433	9
52	.73 455	.81 030	.18 970	.92 425	8
53	.73 474	.81 058	.18 942	.92 416	7
54	.73 494	.81 086	.18 914	.92 408	6
55	.73 513	.81 113	.18 887	.92 400	5
56	.73 533	.81 141	.18 859	.92 392	4
57	.73 552	.81 169	.18 831	.92 384	3
58	.73 572	.81 196	.18 804	.92 376	2
59	.73 591	.81 224	.18 776	.92 367	1
60	.73 611	.81 252	.18 748	.92 359	0
/	θ Lcos	θ Lcot	10 Ltan	θ Lsin	/

33°

/	9 L sin	9 L tan	10 L cot	9 L cos	/
0	.73 611	.81 252	.18 748	.92 359	60
1	.73 630	.81 279	.18 721	.92 351	59
2	.73 650	.81 307	.18 693	.92 343	58
3	.73 669	.81 335	.18 665	.92 335	57
4	.73 689	.81 362	.18 638	.92 326	56
5	.73 708	.81 390	.18 610	.92 318	55
6	.73 727	.81 418	.18 582	.92 310	54
7	.73 747	.81 445	.18 555	.92 302	53
8	.73 766	.81 473	.18 527	.92 293	52
9	.73 785	.81 500	.18 500	.92 285	51
10	.73 805	.81 528	.18 472	.92 277	50
11	.73 824	.81 556	.18 444	.92 269	49
12	.73 843	.81 583	.18 417	.92 260	48
13	.73 863	.81 611	.18 389	.92 252	47
14	.73 882	.81 638	.18 362	.92 244	46
15	.73 901	.81 666	.18 334	.92 235	45
16	.73 921	.81 693	.18 307	.92 227	44
17	.73 940	.81 721	.18 279	.92 219	43
18	.73 959	.81 748	.18 252	.92 211	42
19	.73 978	.81 776	.18 224	.92 202	41
20	.73 997	.81 803	.18 197	.92 194	40
21	.74 017	.81 831	.18 169	.92 186	39
22	.74 036	.81 858	.18 142	.92 177	38
23	.74 055	.81 886	.18 114	.92 169	37
24	.74 074	.81 913	.18 087	.92 161	36
25	.74 093	.81 941	.18 059	.92 152	35
26	.74 113	.81 968	.18 032	.92 144	34
27	.74 132	.81 996	.18 004	.92 136	33
28	.74 151	.82 023	.17 977	.92 127	32
29	.74 170	.82 051	.17 949	.92 119	31
30	.74 189	.82 078	.17 922	.92 111	30
31	.74 208	.82 106	.17 894	.92 102	29
32	.74 227	.82 133	.17 867	.92 094	28
33	.74 246	.82 161	.17 839	.92 086	27
34	.74 265	.82 188	.17 812	.92 077	26
35	.74 284	.82 215	.17 785	.92 069	25
36	.74 303	.82 243	.17 757	.92 060	24
37	.74 322	.82 270	.17 730	.92 052	23
38	.74 341	.82 298	.17 702	.92 044	22
39	.74 360	.82 325	.17 675	.92 035	21
40	.74 379	.82 352	.17 648	.92 027	20
41	.74 398	.82 380	.17 620	.92 018	19
42	.74 417	.82 407	.17 593	.92 010	18
43	.74 436	.82 435	.17 565	.92 002	17
44	.74 455	.82 462	.17 538	.91 993	16
45	.74 474	.82 489	.17 511	.91 985	15
46	.74 493	.82 517	.17 483	.91 976	14
47	.74 512	.82 544	.17 456	.91 968	13
48	.74 531	.82 571	.17 429	.91 959	12
49	.74 549	.82 599	.17 401	.91 951	11
50	.74 568	.82 626	.17 374	.91 942	10
51	.74 587	.82 653	.17 347	.91 934	9
52	.74 606	.82 681	.17 319	.91 925	8
53	.74 625	.82 708	.17 292	.91 917	7
54	.74 644	.82 735	.17 265	.91 908	6
55	.74 662	.82 762	.17 238	.91 900	5
56	.74 681	.82 790	.17 210	.91 891	4
57	.74 700	.82 817	.17 183	.91 883	3
58	.74 719	.82 844	.17 156	.91 874	2
59	.74 737	.82 871	.17 129	.91 866	1
60	.74 756	.82 899	.17 101	.91 857	0
/	9 L cos	9 L cot	10 L tan	9 L sin	/

56°

34°

xxxvii

/	9 L sin	9 L tan	10 L cot	9 L cos	/
0	.74 756	.82 899	.17 101	.91 857	60
1	.74 775	.82 926	.17 074	.91 849	59
2	.74 794	.82 953	.17 047	.91 840	58
3	.74 812	.82 980	.17 020	.91 832	57
4	.74 831	.83 008	.16 992	.91 823	56
5	.74 850	.83 035	.16 965	.91 815	55
6	.74 868	.83 062	.16 938	.91 806	54
7	.74 887	.83 089	.16 911	.91 798	53
8	.74 906	.83 117	.16 883	.91 789	52
9	.74 924	.83 144	.16 856	.91 781	51
10	.74 943	.83 171	.16 829	.91 772	50
11	.74 961	.83 198	.16 802	.91 763	49
12	.74 980	.83 225	.16 775	.91 755	48
13	.74 999	.83 252	.16 748	.91 746	47
14	.75 017	.83 280	.16 720	.91 738	46
15	.75 036	.83 307	.16 693	.91 729	45
16	.75 054	.83 334	.16 666	.91 720	44
17	.75 073	.83 361	.16 639	.91 712	43
18	.75 091	.83 388	.16 612	.91 703	42
19	.75 110	.83 415	.16 585	.91 695	41
20	.75 128	.83 442	.16 558	.91 686	40
21	.75 147	.83 470	.16 530	.91 677	39
22	.75 165	.83 497	.16 503	.91 669	38
23	.75 184	.83 524	.16 476	.91 660	37
24	.75 202	.83 551	.16 449	.91 651	36
25	.75 221	.83 578	.16 422	.91 643	35
26	.75 239	.83 605	.16 395	.91 634	34
27	.75 258	.83 632	.16 368	.91 625	33
28	.75 276	.83 659	.16 341	.91 617	32
29	.75 294	.83 686	.16 314	.91 608	31
30	.75 313	.83 713	.16 287	.91 599	30
31	.75 331	.83 740	.16 260	.91 591	29
32	.75 350	.83 768	.16 232	.91 582	28
33	.75 368	.83 795	.16 205	.91 573	27
34	.75 386	.83 822	.16 178	.91 565	26
35	.75 405	.83 849	.16 151	.91 556	25
36	.75 423	.83 876	.16 124	.91 547	24
37	.75 441	.83 903	.16 097	.91 538	23
38	.75 459	.83 930	.16 070	.91 530	22
39	.75 478	.83 957	.16 043	.91 521	21
40	.75 496	.83 984	.16 016	.91 512	20
41	.75 514	.84 011	.15 989	.91 504	19
42	.75 533	.84 038	.15 962	.91 495	18
43	.75 551	.84 065	.15 935	.91 486	17
44	.75 569	.84 092	.15 908	.91 477	16
45	.75 587	.84 119	.15 881	.91 469	15
46	.75 605	.84 146	.15 854	.91 460	14
47	.75 624	.84 173	.15 827	.91 451	13
48	.75 642	.84 200	.15 800	.91 442	12
49	.75 660	.84 227	.15 773	.91 433	11
50	.75 678	.84 254	.15 746	.91 425	10
51	.75 696	.84 280	.15 720	.91 416	9
52	.75 714	.84 307	.15 693	.91 407	8
53	.75 733	.84 334	.15 666	.91 398	7
54	.75 751	.84 361	.15 639	.91 389	6
55	.75 769	.84 388	.15 612	.91 381	5
56	.75 787	.84 415	.15 585	.91 372	4
57	.75 805	.84 442	.15 558	.91 363	3
58	.75 823	.84 469	.15 531	.91 354	2
59	.75 841	.84 496	.15 504	.91 345	1
60	.75 859	.84 523	.15 477	.91 336	0
/	9 L cos	9 L cot	10 L tan	9 L sin	/

55°

/	9L sin	9L tan	10L cot	9L cos	/
0	.75859	.84523	.15477	.91336	60
1	.75877	.84550	.15450	.91328	59
2	.75895	.84576	.15424	.91319	58
3	.75913	.84603	.15397	.91310	57
4	.75931	.84630	.15370	.91301	56
5	.75949	.84657	.15343	.91292	55
6	.75967	.84684	.15316	.91283	54
7	.75985	.84711	.15289	.91274	53
8	.76003	.84738	.15262	.91266	52
9	.76021	.84764	.15236	.91257	51
10	.76039	.84791	.15209	.91248	50
11	.76057	.84818	.15182	.91239	49
12	.76075	.84845	.15155	.91230	48
13	.76093	.84872	.15128	.91221	47
14	.76111	.84899	.15101	.91212	46
15	.76129	.84925	.15075	.91203	45
16	.76146	.84952	.15048	.91194	44
17	.76164	.84979	.15021	.91185	43
18	.76182	.85006	.14994	.91176	42
19	.76200	.85033	.14967	.91167	41
20	.76218	.85059	.14941	.91158	40
21	.76236	.85086	.14914	.91149	39
22	.76253	.85113	.14887	.91141	38
23	.76271	.85140	.14860	.91132	37
24	.76289	.85166	.14834	.91123	36
25	.76307	.85193	.14807	.91114	35
26	.76324	.85220	.14780	.91105	34
27	.76342	.85247	.14753	.91096	33
28	.76360	.85273	.14727	.91087	32
29	.76378	.85300	.14700	.91078	31
30	.76395	.85327	.14673	.91069	30
31	.76413	.85354	.14646	.91060	29
32	.76431	.85380	.14620	.91051	28
33	.76448	.85407	.14593	.91042	27
34	.76466	.85434	.14566	.91033	26
35	.76484	.85460	.14540	.91023	25
36	.76501	.85487	.14513	.91014	24
37	.76519	.85514	.14486	.91005	23
38	.76537	.85540	.14460	.90996	22
39	.76554	.85567	.14433	.90987	21
40	.76572	.85594	.14406	.90978	20
41	.76590	.85620	.14380	.90969	19
42	.76607	.85647	.14353	.90960	18
43	.76625	.85674	.14326	.90951	17
44	.76642	.85700	.14300	.90942	16
45	.76660	.85727	.14273	.90933	15
46	.76677	.85754	.14246	.90924	14
47	.76695	.85780	.14220	.90915	13
48	.76712	.85807	.14193	.90906	12
49	.76730	.85834	.14166	.90896	11
50	.76747	.85860	.14140	.90887	10
51	.76765	.85887	.14113	.90878	9
52	.76782	.85913	.14087	.90869	8
53	.76800	.85940	.14060	.90860	7
54	.76817	.85967	.14033	.90851	6
55	.76835	.85993	.14007	.90842	5
56	.76852	.86020	.13980	.90832	4
57	.76870	.86046	.13954	.90823	3
58	.76887	.86073	.13927	.90814	2
59	.76904	.86100	.13900	.90805	1
60	.76922	.86126	.13874	.90796	0
/	9L cos	9L cot	10L tan	9L sin	/

/	9L sin	9L tan	10L cot	9L cos	/
0	.76922	.86126	.13874	.90796	60
1	.76939	.86153	.13847	.90787	59
2	.76957	.86179	.13821	.90778	58
3	.76974	.86206	.13794	.90768	57
4	.76991	.86232	.13768	.90759	56
5	.77009	.86259	.13741	.90750	55
6	.77026	.86285	.13715	.90741	54
7	.77043	.86312	.13688	.90731	53
8	.77061	.86338	.13662	.90722	52
9	.77078	.86365	.13635	.90713	51
10	.77095	.86392	.13608	.90704	50
11	.77112	.86418	.13582	.90694	49
12	.77130	.86445	.13555	.90685	48
13	.77147	.86471	.13529	.90676	47
14	.77164	.86498	.13502	.90667	46
15	.77181	.86524	.13476	.90657	45
16	.77199	.86551	.13449	.90648	44
17	.77216	.86577	.13423	.90639	43
18	.77233	.86603	.13397	.90630	42
19	.77250	.86630	.13370	.90620	41
20	.77268	.86656	.13344	.90611	40
21	.77285	.86683	.13317	.90602	39
22	.77302	.86709	.13291	.90592	38
23	.77319	.86736	.13264	.90583	37
24	.77336	.86762	.13238	.90574	36
25	.77353	.86789	.13211	.90565	35
26	.77370	.86815	.13185	.90555	34
27	.77387	.86842	.13158	.90546	33
28	.77405	.86868	.13132	.90537	32
29	.77422	.86894	.13106	.90527	31
30	.77439	.86921	.13079	.90518	30
31	.77456	.86947	.13053	.90509	29
32	.77473	.86974	.13026	.90499	28
33	.77490	.87000	.13000	.90490	27
34	.77507	.87027	.12973	.90480	26
35	.77524	.87053	.12947	.90471	25
36	.77541	.87079	.12921	.90462	24
37	.77558	.87106	.12894	.90452	23
38	.77575	.87132	.12868	.90443	22
39	.77592	.87158	.12842	.90434	21
40	.77609	.87185	.12815	.90424	20
41	.77626	.87211	.12789	.90415	19
42	.77643	.87238	.12762	.90405	18
43	.77660	.87264	.12736	.90396	17
44	.77677	.87290	.12710	.90386	16
45	.77694	.87317	.12683	.90377	15
46	.77711	.87343	.12657	.90368	14
47	.77728	.87369	.12631	.90358	13
48	.77744	.87396	.12604	.90349	12
49	.77761	.87422	.12578	.90339	11
50	.77778	.87448	.12552	.90330	10
51	.77795	.87475	.12525	.90320	9
52	.77812	.87501	.12499	.90311	8
53	.77829	.87527	.12473	.90301	7
54	.77846	.87554	.12446	.90292	6
55	.77862	.87580	.12420	.90282	5
56	.77879	.87606	.12394	.90273	4
57	.77896	.87633	.12367	.90263	3
58	.77913	.87659	.12341	.90254	2
59	.77930	.87685	.12315	.90244	1
60	.77946	.87711	.12289	.90235	0
/	9L cos	9L cot	10L tan	9L sin	/

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/	9Lsin	9Ltan	10Lcot	9Lcos	/
0	.77946	.87711	.12289	.90235	60
1	.77963	.87738	.12262	.90225	59
2	.77980	.87764	.12236	.90216	58
3	.77997	.87790	.12210	.90206	57
4	.78013	.87817	.12183	.90197	56
5	.78030	.87843	.12157	.90187	55
6	.78047	.87869	.12131	.90178	54
7	.78063	.87895	.12105	.90168	53
8	.78080	.87922	.12078	.90159	52
9	.78097	.87948	.12052	.90149	51
10	.78113	.87974	.12026	.90139	50
11	.78130	.88000	.12000	.90130	49
12	.78147	.88027	.11973	.90120	48
13	.78163	.88053	.11947	.90111	47
14	.78180	.88079	.11921	.90101	46
15	.78197	.88105	.11895	.90091	45
16	.78213	.88131	.11869	.90082	44
17	.78230	.88158	.11842	.90072	43
18	.78246	.88184	.11816	.90063	42
19	.78263	.88210	.11790	.90053	41
20	.78280	.88236	.11764	.90043	40
21	.78296	.88262	.11738	.90034	39
22	.78313	.88289	.11711	.90024	38
23	.78329	.88315	.11685	.90014	37
24	.78346	.88341	.11659	.90005	36
25	.78362	.88367	.11633	.99995	35
26	.78379	.88393	.11607	.99985	34
27	.78395	.88420	.11580	.99976	33
28	.78412	.88446	.11554	.99966	32
29	.78428	.88472	.11528	.99956	31
30	.78445	.88498	.11502	.99947	30
31	.78461	.88524	.11476	.99937	29
32	.78478	.88550	.11450	.99927	28
33	.78494	.88577	.11423	.99918	27
34	.78510	.88603	.11397	.99908	26
35	.78527	.88629	.11371	.99898	25
36	.78543	.88655	.11345	.99888	24
37	.78560	.88681	.11319	.99879	23
38	.78576	.88707	.11293	.99869	22
39	.78592	.88733	.11267	.99859	21
40	.78609	.88759	.11241	.99849	20
41	.78625	.88786	.11214	.99840	19
42	.78642	.88812	.11188	.99830	18
43	.78658	.88838	.11162	.99820	17
44	.78674	.88864	.11136	.99810	16
45	.78691	.88890	.11110	.99801	15
46	.78707	.88916	.11084	.99791	14
47	.78723	.88942	.11058	.99781	13
48	.78739	.88968	.11032	.99771	12
49	.78756	.88994	.11006	.99761	11
50	.78772	.89020	.10980	.99752	10
51	.78788	.89046	.10954	.99742	9
52	.78805	.89073	.10927	.99732	8
53	.78821	.89099	.10901	.99722	7
54	.78837	.89125	.10875	.99712	6
55	.78853	.89151	.10849	.99702	5
56	.78869	.89177	.10823	.99693	4
57	.78886	.89203	.10797	.99683	3
58	.78902	.89229	.10771	.99673	2
59	.78918	.89255	.10745	.99663	1
60	.78934	.89281	.10719	.99653	0
/	9Lcos	9Lcot	10Ltan	9Lsin	/

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xxxix

/	9Lsin	9Ltan	10Lcot	9Lcos	/
0	.78934	.89281	.10719	.99653	60
1	.78950	.89307	.10693	.99643	59
2	.78967	.89333	.10667	.99633	58
3	.78983	.89359	.10641	.99624	57
4	.78999	.89385	.10615	.99614	56
5	.79015	.89411	.10589	.99604	55
6	.79031	.89437	.10563	.99594	54
7	.79047	.89463	.10537	.99584	53
8	.79063	.89489	.10511	.99574	52
9	.79079	.89515	.10485	.99564	51
10	.79095	.89541	.10459	.99554	50
11	.79111	.89567	.10433	.99544	49
12	.79128	.89593	.10407	.99534	48
13	.79144	.89619	.10381	.99524	47
14	.79160	.89645	.10355	.99514	46
15	.79176	.89671	.10329	.99504	45
16	.79192	.89697	.10303	.99495	44
17	.79208	.89723	.10277	.99485	43
18	.79224	.89749	.10251	.99475	42
19	.79240	.89775	.10225	.99465	41
20	.79256	.89801	.10199	.99455	40
21	.79272	.89827	.10173	.99445	39
22	.79288	.89853	.10147	.99435	38
23	.79304	.89879	.10121	.99425	37
24	.79319	.89905	.10095	.99415	36
25	.79335	.89931	.10069	.99405	35
26	.79351	.89957	.10043	.99395	34
27	.79367	.89983	.10017	.99385	33
28	.79383	.90009	.99991	.99375	32
29	.79399	.90035	.99965	.99364	31
30	.79415	.90061	.99939	.99354	30
31	.79431	.90086	.99914	.99344	29
32	.79447	.90112	.99888	.99334	28
33	.79463	.90138	.99862	.99324	27
34	.79478	.90164	.99836	.99314	26
35	.79494	.90190	.99810	.99304	25
36	.79510	.90216	.99784	.99294	24
37	.79526	.90242	.99758	.99284	23
38	.79542	.90268	.99732	.99274	22
39	.79558	.90294	.99706	.99264	21
40	.79573	.90320	.99680	.99254	20
41	.79589	.90346	.99654	.99244	19
42	.79605	.90371	.99629	.99233	18
43	.79621	.90397	.99603	.99223	17
44	.79636	.90423	.99577	.99213	16
45	.79652	.90449	.99551	.99203	15
46	.79668	.90475	.99525	.99193	14
47	.79684	.90501	.99499	.99183	13
48	.79699	.90527	.99473	.99173	12
49	.79715	.90553	.99447	.99163	11
50	.79731	.90578	.99422	.99152	10
51	.79746	.90604	.99396	.99142	9
52	.79762	.90630	.99370	.99132	8
53	.79778	.90656	.99344	.99122	7
54	.79793	.90682	.99318	.99112	6
55	.79809	.90708	.99292	.99101	5
56	.79825	.90734	.99266	.99091	4
57	.79840	.90759	.99241	.99081	3
58	.79856	.90785	.99215	.99071	2
59	.79872	.90811	.99189	.99060	1
60	.79887	.90837	.99163	.99050	0
/	9Lcos	9Lcot	10Ltan	9Lsin	/

51°

/	9 L sin	9 L tan	10 L cot	9 L cos	/
0	.79887	.90837	.09163	.89050	60
1	.79903	.90863	.09137	.89040	59
2	.79918	.90889	.09111	.89030	58
3	.79934	.90914	.09086	.89020	57
4	.79950	.90940	.09060	.89009	56
5	.79965	.90966	.09034	.88999	55
6	.79981	.90992	.09008	.88989	54
7	.79996	.91018	.08982	.88978	53
8	.80012	.91043	.08957	.88968	52
9	.80027	.91069	.08931	.88958	51
10	.80043	.91095	.08905	.88948	50
11	.80058	.91121	.08879	.88937	49
12	.80074	.91147	.08853	.88927	48
13	.80089	.91172	.08828	.88917	47
14	.80105	.91198	.08802	.88906	46
15	.80120	.91224	.08776	.88896	45
16	.80136	.91250	.08750	.88886	44
17	.80151	.91276	.08724	.88875	43
18	.80166	.91301	.08699	.88865	42
19	.80182	.91327	.08673	.88855	41
20	.80197	.91353	.08647	.88844	40
21	.80213	.91379	.08621	.88834	39
22	.80228	.91404	.08596	.88824	38
23	.80244	.91430	.08570	.88813	37
24	.80259	.91456	.08544	.88803	36
25	.80274	.91482	.08518	.88793	35
26	.80290	.91507	.08493	.88782	34
27	.80305	.91533	.08467	.88772	33
28	.80320	.91559	.08441	.88761	32
29	.80336	.91585	.08415	.88751	31
30	.80351	.91610	.08390	.88741	30
31	.80366	.91636	.08364	.88730	29
32	.80382	.91662	.08338	.88720	28
33	.80397	.91688	.08312	.88709	27
34	.80412	.91713	.08287	.88699	26
35	.80428	.91739	.08261	.88688	25
36	.80443	.91765	.08235	.88678	24
37	.80458	.91791	.08209	.88668	23
38	.80473	.91816	.08184	.88657	22
39	.80489	.91842	.08158	.88647	21
40	.80504	.91868	.08132	.88636	20
41	.80519	.91893	.08107	.88626	19
42	.80534	.91919	.08081	.88615	18
43	.80550	.91945	.08055	.88605	17
44	.80565	.91971	.08029	.88594	16
45	.80580	.91996	.08004	.88584	15
46	.80595	.92022	.07978	.88573	14
47	.80610	.92048	.07952	.88563	13
48	.80625	.92073	.07927	.88552	12
49	.80641	.92099	.07901	.88542	11
50	.80656	.92125	.07875	.88531	10
51	.80671	.92150	.07850	.88521	9
52	.80686	.92176	.07824	.88510	8
53	.80701	.92202	.07798	.88499	7
54	.80716	.92227	.07773	.88489	6
55	.80731	.92253	.07747	.88478	5
56	.80746	.92279	.07721	.88468	4
57	.80762	.92304	.07696	.88457	3
58	.80777	.92330	.07670	.88447	2
59	.80792	.92356	.07644	.88436	1
60	.80807	.92381	.07619	.88425	0
/	9 L cos	9 L cot	10 L tan	9 L sin	/

/	9 L sin	9 L tan	10 L cot	9 L cos	/
0	.80807	.92381	.07619	.88425	60
1	.80822	.92407	.07593	.88415	59
2	.80837	.92433	.07567	.88404	58
3	.80852	.92458	.07542	.88394	57
4	.80867	.92484	.07516	.88383	56
5	.80882	.92510	.07490	.88372	55
6	.80897	.92535	.07465	.88362	54
7	.80912	.92561	.07439	.88351	53
8	.80927	.92587	.07413	.88340	52
9	.80942	.92612	.07388	.88330	51
10	.80957	.92638	.07362	.88319	50
11	.80972	.92663	.07337	.88308	49
12	.80987	.92689	.07311	.88298	48
13	.81002	.92715	.07285	.88287	47
14	.81017	.92740	.07260	.88276	46
15	.81032	.92766	.07234	.88266	45
16	.81047	.92792	.07208	.88255	44
17	.81061	.92817	.07183	.88244	43
18	.81076	.92843	.07157	.88234	42
19	.81091	.92868	.07132	.88223	41
20	.81106	.92894	.07106	.88212	40
21	.81121	.92920	.07080	.88201	39
22	.81136	.92945	.07055	.88191	38
23	.81151	.92971	.07029	.88180	37
24	.81166	.92996	.07004	.88169	36
25	.81180	.93022	.06978	.88158	35
26	.81195	.93048	.06952	.88148	34
27	.81210	.93073	.06927	.88137	33
28	.81225	.93099	.06901	.88126	32
29	.81240	.93124	.06876	.88115	31
30	.81254	.93150	.06850	.88105	30
31	.81269	.93175	.06825	.88094	29
32	.81284	.93201	.06799	.88083	28
33	.81299	.93227	.06773	.88072	27
34	.81314	.93252	.06748	.88061	26
35	.81328	.93278	.06722	.88051	25
36	.81343	.93303	.06697	.88040	24
37	.81358	.93329	.06671	.88029	23
38	.81372	.93354	.06646	.88018	22
39	.81387	.93380	.06620	.88007	21
40	.81402	.93406	.06594	.87996	20
41	.81417	.93431	.06569	.87985	19
42	.81431	.93457	.06543	.87975	18
43	.81446	.93482	.06518	.87964	17
44	.81461	.93508	.06492	.87953	16
45	.81475	.93533	.06467	.87942	15
46	.81490	.93559	.06441	.87931	14
47	.81505	.93584	.06416	.87920	13
48	.81519	.93610	.06390	.87909	12
49	.81534	.93636	.06364	.87898	11
50	.81549	.93661	.06339	.87887	10
51	.81563	.93687	.06313	.87877	9
52	.81578	.93712	.06288	.87866	8
53	.81592	.93738	.06262	.87855	7
54	.81607	.93763	.06237	.87844	6
55	.81622	.93789	.06211	.87833	5
56	.81636	.93814	.06186	.87822	4
57	.81651	.93840	.06160	.87811	3
58	.81665	.93865	.06135	.87800	2
59	.81680	.93891	.06109	.87789	1
60	.81694	.93916	.06084	.87778	0
/	9 L cos	9 L cot	10 L tan	9 L sin	/

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/	9 L sin	9 L tan	10 L cot	9 L cos	/
0	.81694	.93916	.06084	.87778	60
1	.81709	.93942	.06058	.87767	59
2	.81723	.93967	.06033	.87756	58
3	.81738	.93993	.06007	.87745	57
4	.81752	.94018	.05982	.87734	56
5	.81767	.94044	.05956	.87723	55
6	.81781	.94069	.05931	.87712	54
7	.81796	.94095	.05905	.87701	53
8	.81810	.94120	.05880	.87690	52
9	.81825	.94146	.05854	.87679	51
10	.81839	.94171	.05829	.87668	50
11	.81854	.94197	.05803	.87657	49
12	.81868	.94222	.05778	.87646	48
13	.81882	.94248	.05752	.87635	47
14	.81897	.94273	.05727	.87624	46
15	.81911	.94299	.05701	.87613	45
16	.81926	.94324	.05676	.87601	44
17	.81940	.94350	.05650	.87590	43
18	.81955	.94375	.05625	.87579	42
19	.81969	.94401	.05599	.87568	41
20	.81983	.94426	.05574	.87557	40
21	.81998	.94452	.05548	.87546	39
22	.82012	.94477	.05523	.87535	38
23	.82026	.94503	.05497	.87524	37
24	.82041	.94528	.05472	.87513	36
25	.82055	.94554	.05446	.87501	35
26	.82069	.94579	.05421	.87490	34
27	.82084	.94604	.05396	.87479	33
28	.82098	.94630	.05370	.87468	32
29	.82112	.94655	.05345	.87457	31
30	.82126	.94681	.05319	.87446	30
31	.82141	.94706	.05294	.87434	29
32	.82155	.94732	.05268	.87423	28
33	.82169	.94757	.05243	.87412	27
34	.82184	.94783	.05217	.87401	26
35	.82198	.94808	.05192	.87390	25
36	.82212	.94834	.05166	.87378	24
37	.82226	.94859	.05141	.87367	23
38	.82240	.94884	.05116	.87356	22
39	.82255	.94910	.05090	.87345	21
40	.82269	.94935	.05065	.87334	20
41	.82283	.94961	.05039	.87322	19
42	.82297	.94986	.05014	.87311	18
43	.82311	.95012	.04988	.87300	17
44	.82326	.95037	.04963	.87288	16
45	.82340	.95062	.04938	.87277	15
46	.82354	.95088	.04912	.87266	14
47	.82368	.95113	.04887	.87255	13
48	.82382	.95139	.04861	.87243	12
49	.82396	.95164	.04836	.87232	11
50	.82410	.95190	.04810	.87221	10
51	.82424	.95215	.04785	.87209	9
52	.82439	.95240	.04760	.87198	8
53	.82453	.95266	.04734	.87187	7
54	.82467	.95291	.04709	.87175	6
55	.82481	.95317	.04683	.87164	5
56	.82495	.95342	.04658	.87153	4
57	.82509	.95368	.04632	.87141	3
58	.82523	.95393	.04607	.87130	2
59	.82537	.95418	.04582	.87119	1
60	.82551	.95444	.04556	.87107	0
/	9 L cos	9 L cot	10 L tan	9 L sin	/

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42°

xli

/	9 L sin	9 L tan	10 L cot	9 L cos	/
0	.82551	.95444	.04556	.87107	60
1	.82565	.95469	.04531	.87096	59
2	.82579	.95495	.04505	.87085	58
3	.82593	.95520	.04480	.87073	57
4	.82607	.95545	.04455	.87062	56
5	.82621	.95571	.04429	.87050	55
6	.82635	.95596	.04404	.87039	54
7	.82649	.95622	.04378	.87028	53
8	.82663	.95647	.04353	.87016	52
9	.82677	.95672	.04328	.87005	51
10	.82691	.95698	.04302	.86993	50
11	.82705	.95723	.04277	.86982	49
12	.82719	.95748	.04252	.86970	48
13	.82733	.95774	.04226	.86959	47
14	.82747	.95799	.04201	.86947	46
15	.82761	.95825	.04175	.86936	45
16	.82775	.95850	.04150	.86924	44
17	.82788	.95875	.04125	.86913	43
18	.82802	.95901	.04099	.86902	42
19	.82816	.95926	.04074	.86890	41
20	.82830	.95952	.04048	.86879	40
21	.82844	.95977	.04023	.86867	39
22	.82858	.96002	.03998	.86855	38
23	.82872	.96028	.03972	.86844	37
24	.82885	.96053	.03947	.86832	36
25	.82899	.96078	.03922	.86821	35
26	.82913	.96104	.03896	.86809	34
27	.82927	.96129	.03871	.86798	33
28	.82941	.96155	.03845	.86786	32
29	.82955	.96180	.03820	.86775	31
30	.82968	.96205	.03795	.86763	30
31	.82982	.96231	.03769	.86752	29
32	.82996	.96256	.03744	.86740	28
33	.83010	.96281	.03719	.86728	27
34	.83023	.96307	.03693	.86717	26
35	.83037	.96332	.03668	.86705	25
36	.83051	.96357	.03643	.86694	24
37	.83065	.96383	.03617	.86682	23
38	.83078	.96408	.03592	.86670	22
39	.83092	.96433	.03567	.86659	21
40	.83106	.96459	.03541	.86647	20
41	.83120	.96484	.03516	.86635	19
42	.83133	.96510	.03490	.86624	18
43	.83147	.96535	.03465	.86612	17
44	.83161	.96560	.03440	.86600	16
45	.83174	.96586	.03414	.86589	15
46	.83188	.96611	.03389	.86577	14
47	.83202	.96636	.03364	.86565	13
48	.83215	.96662	.03338	.86554	12
49	.83229	.96687	.03313	.86542	11
50	.83242	.96712	.03288	.86530	10
51	.83256	.96738	.03262	.86518	9
52	.83270	.96763	.03237	.86507	8
53	.83283	.96788	.03212	.86495	7
54	.83297	.96814	.03186	.86483	6
55	.83310	.96839	.03161	.86472	5
56	.83324	.96864	.03136	.86460	4
57	.83338	.96890	.03110	.86448	3
58	.83351	.96915	.03085	.86436	2
59	.83365	.96940	.03060	.86425	1
60	.83378	.96966	.03034	.86413	0
/	9 L cos	9 L cot	10 L tan	9 L sin	/

47°

/	9L sin	9L tan	10L cot	9L cos	/
0	.83378	.96966	.03034	.86413	60
1	.83392	.96991	.03009	.86401	59
2	.83405	.97016	.02984	.86389	58
3	.83419	.97042	.02958	.86377	57
4	.83432	.97067	.02933	.86366	56
5	.83446	.97092	.02908	.86354	55
6	.83459	.97118	.02882	.86342	54
7	.83473	.97143	.02857	.86330	53
8	.83486	.97168	.02832	.86318	52
9	.83500	.97193	.02807	.86306	51
10	.83513	.97219	.02781	.86295	50
11	.83527	.97244	.02756	.86283	49
12	.83540	.97269	.02731	.86271	48
13	.83554	.97295	.02705	.86259	47
14	.83567	.97320	.02680	.86247	46
15	.83581	.97345	.02655	.86235	45
16	.83594	.97371	.02629	.86223	44
17	.83608	.97396	.02604	.86211	43
18	.83621	.97421	.02579	.86200	42
19	.83634	.97447	.02553	.86188	41
20	.83648	.97472	.02528	.86176	40
21	.83661	.97497	.02503	.86164	39
22	.83674	.97523	.02477	.86152	38
23	.83688	.97548	.02452	.86140	37
24	.83701	.97573	.02427	.86128	36
25	.83715	.97598	.02402	.86116	35
26	.83728	.97624	.02376	.86104	34
27	.83741	.97649	.02351	.86092	33
28	.83755	.97674	.02326	.86080	32
29	.83768	.97700	.02300	.86068	31
30	.83781	.97725	.02275	.86056	30
31	.83795	.97750	.02250	.86044	29
32	.83808	.97776	.02224	.86032	28
33	.83821	.97801	.02199	.86020	27
34	.83834	.97826	.02174	.86008	26
35	.83848	.97851	.02149	.85996	25
36	.83861	.97877	.02123	.85984	24
37	.83874	.97902	.02098	.85972	23
38	.83887	.97927	.02073	.85960	22
39	.83901	.97953	.02047	.85948	21
40	.83914	.97978	.02022	.85936	20
41	.83927	.98003	.01997	.85924	19
42	.83940	.98029	.01971	.85912	18
43	.83954	.98054	.01946	.85900	17
44	.83967	.98079	.01921	.85888	16
45	.83980	.98104	.01896	.85876	15
46	.83993	.98130	.01870	.85864	14
47	.84006	.98155	.01845	.85851	13
48	.84020	.98180	.01820	.85839	12
49	.84033	.98206	.01794	.85827	11
50	.84046	.98231	.01769	.85815	10
51	.84059	.98256	.01744	.85803	9
52	.84072	.98281	.01719	.85791	8
53	.84085	.98307	.01693	.85779	7
54	.84098	.98332	.01668	.85766	6
55	.84112	.98357	.01643	.85754	5
56	.84125	.98383	.01617	.85742	4
57	.84138	.98408	.01592	.85730	3
58	.84151	.98433	.01567	.85718	2
59	.84164	.98458	.01542	.85706	1
60	.84177	.98484	.01516	.85693	0
/	9L cos	9L cot	10L tan	9L sin	/

/	9L sin	9L tan	10L cot	9L cos	/
0	.84177	.98484	.01516	.85693	60
1	.84190	.98509	.01491	.85681	59
2	.84203	.98534	.01466	.85669	58
3	.84216	.98560	.01440	.85657	57
4	.84229	.98585	.01415	.85645	56
5	.84242	.98610	.01390	.85632	55
6	.84255	.98635	.01365	.85620	54
7	.84269	.98661	.01339	.85608	53
8	.84282	.98686	.01314	.85596	52
9	.84295	.98711	.01289	.85583	51
10	.84308	.98737	.01263	.85571	50
11	.84321	.98762	.01238	.85559	49
12	.84334	.98787	.01213	.85547	48
13	.84347	.98812	.01188	.85534	47
14	.84360	.98838	.01162	.85522	46
15	.84373	.98863	.01137	.85510	45
16	.84385	.98888	.01112	.85497	44
17	.84398	.98913	.01087	.85485	43
18	.84411	.98939	.01061	.85473	42
19	.84424	.98964	.01036	.85460	41
20	.84437	.98989	.01011	.85448	40
21	.84450	.99015	.00985	.85436	39
22	.84463	.99040	.00960	.85423	38
23	.84476	.99065	.00935	.85411	37
24	.84489	.99090	.00910	.85399	36
25	.84502	.99116	.00884	.85386	35
26	.84515	.99141	.00859	.85374	34
27	.84528	.99166	.00834	.85361	33
28	.84540	.99191	.00809	.85349	32
29	.84553	.99217	.00783	.85337	31
30	.84566	.99242	.00758	.85324	30
31	.84579	.99267	.00733	.85312	29
32	.84592	.99293	.00707	.85299	28
33	.84605	.99318	.00682	.85287	27
34	.84618	.99343	.00657	.85274	26
35	.84630	.99368	.00632	.85262	25
36	.84643	.99394	.00606	.85250	24
37	.84656	.99419	.00581	.85237	23
38	.84669	.99444	.00556	.85225	22
39	.84682	.99469	.00531	.85212	21
40	.84694	.99495	.00505	.85200	20
41	.84707	.99520	.00480	.85187	19
42	.84720	.99545	.00455	.85175	18
43	.84733	.99570	.00430	.85162	17
44	.84745	.99596	.00404	.85150	16
45	.84758	.99621	.00379	.85137	15
46	.84771	.99646	.00354	.85125	14
47	.84784	.99672	.00328	.85112	13
48	.84796	.99697	.00303	.85100	12
49	.84809	.99722	.00278	.85087	11
50	.84822	.99747	.00253	.85074	10
51	.84835	.99773	.00227	.85062	9
52	.84847	.99798	.00202	.85049	8
53	.84860	.99823	.00177	.85037	7
54	.84873	.99848	.00152	.85024	6
55	.84885	.99874	.00126	.85012	5
56	.84898	.99899	.00101	.85000	4
57	.84911	.99924	.00076	.84986	3
58	.84923	.99949	.00051	.84974	2
59	.84936	.99975	.00025	.84961	1
60	.84949	.00000	.00000	.84949	0
/	9L cos	10L cot	10L tan	9L sin	/

FIVE PLACE
LOGARITHMIC AND NATURAL HAVERSINES
FOR EACH TEN MINUTES OF ARC

Pages xlv-xlviii

°	Log hav	Nat hav	°	Log hav	Nat hav	°	Log hav	Nat hav	°	Log hav	Nat hav
0	—	.00000	10	7.88 059	.00 760	20	8.47 934	.03 015	30	8.82 599	.06 699
10	4.32 539	.00000	10	7.89 491	.00 785	10	8.48 648	.03 065	10	8.83 069	.06 772
20	4.92 745	.00001	20	7.90 900	.00 811	20	8.49 355	.03 116	20	8.83 537	.06 845
30	5.27 963	.00002	30	7.92 286	.00 837	30	8.50 056	.03 166	30	8.84 002	.06 919
40	5.52 951	.00003	40	7.93 650	.00 864	40	8.50 752	.03 218	40	8.84 464	.06 993
50	5.72 332	.00005	50	7.94 992	.00 891	50	8.51 442	.03 269	50	8.84 923	.07 067
1	5.88 168	.00008	11	7.96 315	.00 919	21	8.52 127	.03 321	31	8.85 380	.07 142
10	6.01 557	.00010	10	7.97 617	.00 947	10	8.52 806	.03 373	10	8.85 834	.07 217
20	6.13 155	.00014	20	7.98 899	.00 975	20	8.53 479	.03 426	20	8.86 286	.07 292
30	6.23 385	.00017	30	8.00 163	.01 004	30	8.54 147	.03 479	30	8.86 735	.07 368
40	6.32 536	.00021	40	8.01 409	.01 033	40	8.54 810	.03 533	40	8.87 182	.07 444
50	6.40 814	.00026	50	8.02 637	.01 063	50	8.55 467	.03 587	50	8.87 626	.07 521
2	6.48 371	.00030	12	8.03 847	.01 093	22	8.56 120	.03 641	32	8.88 068	.07 598
10	6.55 323	.00036	10	8.05 041	.01 123	10	8.56 767	.03 695	10	8.88 507	.07 675
20	6.61 759	.00041	20	8.06 218	.01 154	20	8.57 410	.03 751	20	8.88 944	.07 752
30	6.67 751	.00048	30	8.07 379	.01 185	30	8.58 047	.03 806	30	8.89 379	.07 830
40	6.73 355	.00054	40	8.08 525	.01 217	40	8.58 680	.03 862	40	8.89 811	.07 909
50	6.78 620	.00061	50	8.09 656	.01 249	50	8.59 308	.03 918	50	8.90 241	.07 987
3	6.83 584	.00069	13	8.10 772	.01 282	23	8.59 931	.03 975	33	8.90 668	.08 066
10	6.88 279	.00076	10	8.11 873	.01 314	10	8.60 550	.04 032	10	8.91 094	.08 146
20	6.92 733	.00085	20	8.12 961	.01 348	20	8.61 164	.04 089	20	8.91 517	.08 226
30	6.96 970	.00093	30	8.14 035	.01 382	30	8.61 773	.04 147	30	8.91 938	.08 306
40	7.01 009	.00102	40	8.15 096	.01 416	40	8.62 379	.04 205	40	8.92 356	.08 386
50	7.04 869	.00112	50	8.16 144	.01 450	50	8.62 979	.04 264	50	8.92 773	.08 467
4	7.08 564	.00122	14	8.17 179	.01 485	24	8.63 576	.04 323	34	8.93 187	.08 548
10	7.12 108	.00132	10	8.18 202	.01 521	10	8.64 168	.04 382	10	8.93 599	.08 630
20	7.15 513	.00143	20	8.19 212	.01 556	20	8.64 756	.04 442	20	8.94 009	.08 711
30	7.18 790	.00154	30	8.20 211	.01 593	30	8.65 340	.04 502	30	8.94 417	.08 794
40	7.21 947	.00166	40	8.21 199	.01 629	40	8.65 920	.04 562	40	8.94 823	.08 876
50	7.24 993	.00178	50	8.22 175	.01 666	50	8.66 496	.04 623	50	8.95 227	.08 959
5	7.27 936	.00190	15	8.23 140	.01 704	25	8.67 067	.04 685	35	8.95 628	.09 042
10	7.30 782	.00203	10	8.24 094	.01 742	10	8.67 635	.04 746	10	8.96 028	.09 126
20	7.33 538	.00216	20	8.25 037	.01 780	20	8.68 199	.04 808	20	8.96 426	.09 210
30	7.36 209	.00230	30	8.25 971	.01 818	30	8.68 760	.04 871	30	8.96 821	.09 294
40	7.38 800	.00244	40	8.26 894	.01 858	40	8.69 316	.04 934	40	8.97 215	.09 379
50	7.41 315	.00259	50	8.27 807	.01 897	50	8.69 869	.04 997	50	8.97 607	.09 464
6	7.43 760	.00274	16	8.28 711	.01 937	26	8.70 418	.05 060	36	8.97 997	.09 549
10	7.46 138	.00289	10	8.29 605	.01 977	10	8.70 963	.05 124	10	8.98 384	.09 635
20	7.48 452	.00305	20	8.30 490	.02 018	20	8.71 505	.05 189	20	8.98 770	.09 721
30	7.50 706	.00321	30	8.31 366	.02 059	30	8.72 043	.05 253	30	8.99 154	.09 807
40	7.52 902	.00338	40	8.32 233	.02 101	40	8.72 578	.05 318	40	8.99 536	.09 894
50	7.55 045	.00355	50	8.33 091	.02 142	50	8.73 109	.05 384	50	8.99 917	.09 981
7	7.57 135	.00373	17	8.33 940	.02 185	27	8.73 637	.05 450	37	9.00 295	.10 068
10	7.59 176	.00391	10	8.34 782	.02 227	10	8.74 162	.05 516	10	9.00 672	.10 156
20	7.61 170	.00409	20	8.35 614	.02 271	20	8.74 683	.05 582	20	9.01 047	.10 244
30	7.63 120	.00428	30	8.36 439	.02 314	30	8.75 201	.05 649	30	9.01 420	.10 332
40	7.65 026	.00447	40	8.37 256	.02 358	40	8.75 715	.05 717	40	9.01 791	.10 421
50	7.66 891	.00467	50	8.38 065	.02 402	50	8.76 227	.05 785	50	9.02 161	.10 510
8	7.68 717	.00487	18	8.38 867	.02 447	28	8.76 735	.05 853	38	9.02 528	.10 599
10	7.70 505	.00507	10	8.39 660	.02 492	10	8.77 240	.05 921	10	9.02 894	.10 689
20	7.72 257	.00528	20	8.40 447	.02 538	20	8.77 742	.05 990	20	9.03 259	.10 779
30	7.73 974	.00549	30	8.41 226	.02 584	30	8.78 241	.06 059	30	9.03 621	.10 870
40	7.75 657	.00571	40	8.41 998	.02 630	40	8.78 737	.06 129	40	9.03 982	.10 960
50	7.77 308	.00593	50	8.42 764	.02 677	50	8.79 230	.06 199	50	9.04 341	.11 051
9	7.78 929	.00616	19	8.43 522	.02 724	29	8.79 720	.06 269	39	9.04 699	.11 143
10	7.80 519	.00639	10	8.44 273	.02 772	10	8.80 207	.06 340	10	9.05 055	.11 234
20	7.82 081	.00662	20	8.45 018	.02 820	20	8.80 691	.06 411	20	9.05 409	.11 326
30	7.83 615	.00686	30	8.45 757	.02 868	30	8.81 172	.06 482	30	9.05 762	.11 419
40	7.85 122	.00710	40	8.46 489	.02 917	40	8.81 651	.06 554	40	9.06 113	.11 511
50	7.86 603	.00735	50	8.47 215	.02 966	50	8.82 126	.06 626	50	9.06 462	.11 604
10	7.88 059	.00760	20	8.47 934	.03 015	30	8.82 599	.06 699	40	9.06 810	.11 698

°	9 L hav	Nath hav	°	9 L hav	Nath hav	°	9 L hav	Nath hav	°	9 L hav	Nath hav
40	.06 810	.11 698	50	.25 190	.17 861	60	.39 794	.25 000	70	.51 718	.32 899
10	.07 157	.11 791	10	.25 460	.17 972	10	.40 012	.25 126	10	.51 898	.33 036
20	.07 501	.11 885	20	.25 729	.18 084	20	.40 230	.25 252	20	.52 078	.33 173
30	.07 845	.11 980	30	.25 998	.18 196	30	.40 447	.25 379	30	.52 257	.33 310
40	.08 186	.12 074	40	.26 265	.18 308	40	.40 663	.25 506	40	.52 436	.33 447
50	.08 526	.12 169	50	.26 532	.18 421	50	.40 879	.25 632	50	.52 613	.33 584
41	.08 865	.12 265	51	.26 797	.18 534	61	.41 094	.25 760	71	.52 791	.33 722
10	.09 202	.12 360	10	.27 061	.18 647	10	.41 308	.25 887	10	.52 968	.33 859
20	.09 538	.12 456	20	.27 325	.18 761	20	.41 521	.26 014	20	.53 144	.33 997
30	.09 872	.12 552	30	.27 587	.18 874	30	.41 734	.26 142	30	.53 320	.34 135
40	.10 205	.12 649	40	.27 848	.18 988	40	.41 946	.26 270	40	.53 495	.34 273
50	.10 536	.12 746	50	.28 109	.19 102	50	.42 157	.26 398	50	.53 670	.34 411
42	.10 866	.12 843	52	.28 368	.19 217	62	.42 368	.26 526	72	.53 844	.34 549
10	.11 194	.12 940	10	.28 627	.19 332	10	.42 578	.26 655	10	.54 017	.34 688
20	.11 521	.13 038	20	.28 885	.19 447	20	.42 787	.26 784	20	.54 190	.34 826
30	.11 847	.13 136	30	.29 141	.19 562	30	.42 996	.26 913	30	.54 363	.34 965
40	.12 171	.13 235	40	.29 397	.19 677	40	.43 203	.27 042	40	.54 535	.35 103
50	.12 494	.13 333	50	.29 652	.19 793	50	.43 411	.27 171	50	.54 707	.35 242
43	.12 815	.13 432	53	.29 906	.19 909	63	.43 617	.27 300	73	.54 878	.35 381
10	.13 135	.13 532	10	.30 158	.20 026	10	.43 823	.27 430	10	.55 048	.35 521
20	.13 454	.13 631	20	.30 410	.20 142	20	.44 028	.27 560	20	.55 218	.35 660
30	.13 771	.13 731	30	.30 662	.20 259	30	.44 232	.27 690	30	.55 387	.35 799
40	.14 087	.13 832	40	.30 912	.20 376	40	.44 436	.27 820	40	.55 556	.35 939
50	.14 402	.13 932	50	.31 161	.20 493	50	.44 639	.27 951	50	.55 725	.36 078
44	.14 715	.14 033	54	.31 409	.20 611	64	.44 842	.28 081	74	.55 893	.36 218
10	.15 027	.14 134	10	.31 657	.20 729	10	.45 044	.28 212	10	.56 060	.36 358
20	.15 338	.14 236	20	.31 903	.20 847	20	.45 245	.28 343	20	.56 227	.36 498
30	.15 647	.14 337	30	.32 149	.20 965	30	.45 446	.28 474	30	.56 393	.36 638
40	.15 955	.14 440	40	.32 394	.21 083	40	.45 645	.28 606	40	.56 559	.36 778
50	.16 262	.14 542	50	.32 638	.21 202	50	.45 845	.28 737	50	.56 725	.36 919
45	.16 568	.14 645	55	.32 881	.21 321	65	.46 043	.28 869	75	.56 889	.37 059
10	.16 872	.14 748	10	.33 123	.21 440	10	.46 241	.29 001	10	.57 054	.37 200
20	.17 175	.14 851	20	.33 365	.21 560	20	.46 439	.29 133	20	.57 218	.37 340
30	.17 477	.14 955	30	.33 605	.21 680	30	.46 635	.29 265	30	.57 381	.37 481
40	.17 778	.15 058	40	.33 845	.21 800	40	.46 831	.29 398	40	.57 544	.37 622
50	.18 077	.15 163	50	.34 084	.21 920	50	.47 027	.29 530	50	.57 706	.37 763
46	.18 376	.15 267	56	.34 322	.22 040	66	.47 222	.29 663	76	.57 868	.37 904
10	.18 673	.15 372	10	.34 559	.22 161	10	.47 416	.29 796	10	.58 030	.38 045
20	.18 968	.15 477	20	.34 795	.22 282	20	.47 610	.29 929	20	.58 191	.38 186
30	.19 263	.15 582	30	.35 031	.22 403	30	.47 803	.30 063	30	.58 351	.38 328
40	.19 557	.15 688	40	.35 266	.22 525	40	.47 995	.30 196	40	.58 511	.38 469
50	.19 849	.15 794	50	.35 500	.22 646	50	.48 187	.30 330	50	.58 671	.38 611
47	.20 140	.15 900	57	.35 733	.22 768	67	.48 378	.30 463	77	.58 830	.38 752
10	.20 430	.16 007	10	.35 965	.22 890	10	.48 568	.30 597	10	.58 989	.38 894
20	.20 719	.16 113	20	.36 196	.23 012	20	.48 758	.30 732	20	.59 147	.39 036
30	.21 006	.16 220	30	.36 427	.23 135	30	.48 948	.30 866	30	.59 304	.39 178
40	.21 293	.16 328	40	.36 657	.23 258	40	.49 137	.31 000	40	.59 461	.39 320
50	.21 578	.16 436	50	.36 886	.23 381	50	.49 325	.31 135	50	.59 618	.39 462
48	.21 863	.16 543	58	.37 114	.23 504	68	.49 512	.31 270	78	.59 774	.39 604
10	.22 146	.16 652	10	.37 342	.23 627	10	.49 699	.31 405	10	.59 930	.39 746
20	.22 428	.16 760	20	.37 569	.23 751	20	.49 886	.31 540	20	.60 085	.39 889
30	.22 709	.16 869	30	.37 794	.23 875	30	.50 072	.31 675	30	.60 240	.40 032
40	.22 989	.16 978	40	.38 020	.23 999	40	.50 257	.31 810	40	.60 395	.40 174
50	.23 268	.17 087	50	.38 244	.24 124	50	.50 442	.31 946	50	.60 549	.40 317
49	.23 545	.17 197	59	.38 468	.24 248	69	.50 626	.32 082	79	.60 702	.40 460
10	.23 822	.17 307	10	.38 691	.24 373	10	.50 809	.32 217	10	.60 855	.40 602
20	.24 098	.17 417	20	.38 913	.24 498	20	.50 992	.32 353	20	.61 008	.40 745
30	.24 372	.17 528	30	.39 134	.24 623	30	.51 174	.32 490	30	.61 160	.40 888
40	.24 646	.17 638	40	.39 355	.24 749	40	.51 356	.32 626	40	.61 312	.41 031
50	.24 918	.17 749	50	.39 575	.24 874	50	.51 538	.32 762	50	.61 463	.41 174
50	.25 190	.17 861	60	.39 794	.25 000	70	.51 718	.32 899	80	.61 614	.41 318

°	′	°L hav	Nathav	°	′	°L hav	Nathav	°	′	°L hav	Nathav	°	′	°L hav	Nathav
80		.61 614	.41 318	90		.69 897	.50 000	100		.76 851	.58 682	110		.82 673	.67 101
10		.61 764	.41 461	10		.70 023	.50 145	10		.76 957	.58 826	10		.82 761	.67 238
20		.61 914	.41 604	20		.70 149	.50 291	20		.77 062	.58 969	20		.82 849	.67 374
30		.62 063	.41 748	30		.70 274	.50 436	30		.77 167	.59 112	30		.82 937	.67 510
40		.62 212	.41 891	40		.70 399	.50 582	40		.77 272	.59 255	40		.83 025	.67 647
50		.62 361	.42 035	50		.70 524	.50 727	50		.77 377	.59 398	50		.83 112	.67 783
81		.62 509	.42 178	91		.70 648	.50 873	101		.77 481	.59 540	111		.83 199	.67 918
10		.62 657	.42 322	10		.70 772	.51 018	10		.77 585	.59 683	10		.83 285	.68 054
20		.62 804	.42 466	20		.70 896	.51 163	20		.77 689	.59 826	20		.83 372	.68 190
30		.62 951	.42 610	30		.71 019	.51 309	30		.77 792	.59 968	30		.83 458	.68 325
40		.63 097	.42 753	40		.71 142	.51 454	40		.77 895	.60 111	40		.83 544	.68 460
50		.63 243	.42 897	50		.71 265	.51 600	50		.77 998	.60 253	50		.83 630	.68 595
82		.63 389	.43 041	92		.71 387	.51 745	102		.78 101	.60 396	112		.83 715	.68 730
10		.63 534	.43 185	10		.71 509	.51 890	10		.78 203	.60 538	10		.83 800	.68 865
20		.63 678	.43 330	20		.71 630	.52 036	20		.78 305	.60 680	20		.83 885	.69 000
30		.63 823	.43 474	30		.71 751	.52 181	30		.78 406	.60 822	30		.83 969	.69 134
40		.63 966	.43 618	40		.71 872	.52 326	40		.78 507	.60 964	40		.84 054	.69 268
50		.64 110	.43 762	50		.71 992	.52 472	50		.78 608	.61 106	50		.84 138	.69 403
83		.64 253	.43 907	93		.72 112	.52 617	103		.78 709	.61 248	113		.84 221	.69 537
10		.64 396	.44 051	10		.72 232	.52 762	10		.78 809	.61 389	10		.84 305	.69 670
20		.64 538	.44 195	20		.72 352	.52 907	20		.78 909	.61 531	20		.84 388	.69 804
30		.64 679	.44 340	30		.72 471	.53 052	30		.79 009	.61 672	30		.84 471	.69 937
40		.64 821	.44 484	40		.72 589	.53 198	40		.79 108	.61 814	40		.84 554	.70 071
50		.64 962	.44 629	50		.72 708	.53 343	50		.79 208	.61 955	50		.84 636	.70 204
84		.65 102	.44 774	94		.72 825	.53 488	104		.79 306	.62 096	114		.84 718	.70 337
10		.65 242	.44 918	10		.72 943	.53 633	10		.79 405	.62 237	10		.84 800	.70 470
20		.65 382	.45 063	20		.73 060	.53 778	20		.79 503	.62 378	20		.84 882	.70 602
30		.65 521	.45 208	30		.73 177	.53 923	30		.79 601	.62 519	30		.84 963	.70 735
40		.65 660	.45 353	40		.73 294	.54 068	40		.79 699	.62 660	40		.85 044	.70 867
50		.65 799	.45 497	50		.73 410	.54 213	50		.79 796	.62 800	50		.85 125	.70 999
85		.65 937	.45 642	95		.73 526	.54 358	105		.79 893	.62 941	115		.85 206	.71 131
10		.66 074	.45 787	10		.73 642	.54 503	10		.79 990	.63 081	10		.85 286	.71 263
20		.66 212	.45 932	20		.73 757	.54 647	20		.80 087	.63 222	20		.85 366	.71 394
30		.66 348	.46 077	30		.73 872	.54 792	30		.80 183	.63 362	30		.85 446	.71 526
40		.66 485	.46 222	40		.73 987	.54 937	40		.80 279	.63 502	40		.85 526	.71 657
50		.66 621	.46 367	50		.74 101	.55 082	50		.80 374	.63 642	50		.85 605	.71 788
86		.66 757	.46 512	96		.74 215	.55 226	106		.80 470	.63 782	116		.85 684	.71 919
10		.66 892	.46 657	10		.74 328	.55 371	10		.80 565	.63 922	10		.85 763	.72 049
20		.67 027	.46 802	20		.74 442	.55 516	20		.80 660	.64 061	20		.85 841	.72 180
30		.67 161	.46 948	30		.74 554	.55 660	30		.80 754	.64 201	30		.85 920	.72 310
40		.67 295	.47 093	40		.74 667	.55 805	40		.80 848	.64 340	40		.85 998	.72 440
50		.67 429	.47 238	50		.74 779	.55 949	50		.80 942	.64 479	50		.86 076	.72 570
87		.67 562	.47 383	97		.74 891	.56 093	107		.81 036	.64 619	117		.86 153	.72 700
10		.67 695	.47 528	10		.75 003	.56 238	10		.81 129	.64 758	10		.86 230	.72 829
20		.67 828	.47 674	20		.75 114	.56 382	20		.81 222	.64 897	20		.86 307	.72 958
30		.67 960	.47 819	30		.75 225	.56 526	30		.81 315	.65 035	30		.86 384	.73 087
40		.68 092	.47 964	40		.75 336	.56 670	40		.81 407	.65 174	40		.86 461	.73 216
50		.68 223	.48 110	50		.75 446	.56 815	50		.81 500	.65 312	50		.86 537	.73 345
88		.68 354	.48 255	98		.75 556	.56 959	108		.81 592	.65 451	118		.86 613	.73 474
10		.68 485	.48 400	10		.75 666	.57 103	10		.81 683	.65 589	10		.86 689	.73 602
20		.68 615	.48 546	20		.75 775	.57 247	20		.81 775	.65 727	20		.86 764	.73 730
30		.68 745	.48 691	30		.75 884	.57 390	30		.81 866	.65 865	30		.86 840	.73 858
40		.68 875	.48 837	40		.75 993	.57 534	40		.81 956	.66 003	40		.86 915	.73 986
50		.69 004	.48 982	50		.76 101	.57 678	50		.82 047	.66 141	50		.86 990	.74 113
89		.69 132	.49 127	99		.76 209	.57 822	109		.82 137	.66 278	119		.87 064	.74 240
10		.69 261	.49 273	10		.76 317	.57 965	10		.82 227	.66 416	10		.87 138	.74 368
20		.69 389	.49 418	20		.76 424	.58 109	20		.82 317	.66 553	20		.87 212	.74 494
30		.69 516	.49 564	30		.76 531	.58 252	30		.82 406	.66 690	30		.87 286	.74 621
40		.69 644	.49 709	40		.76 638	.58 396	40		.82 495	.66 827	40		.87 360	.74 748
50		.69 770	.49 855	50		.76 745	.58 539	50		.82 584	.66 964	50		.87 433	.74 874
90		.69 897	.50 000	100		.76 851	.58 682	110		.82 673	.67 101	120		.87 506	.75 000

° /	° L hav Nat hav	° /	° L hav Nat hav	° /	° L hav Nat hav	° /	° L hav Nat hav
120	.87 506 .75 000	130	.91 455 .82 139	140	.94 597 .88 302	150	.96 989 .93 301
10	.87 579 .75 126	10	.91 514 .82 251	10	.94 643 .88 396	10	.97 022 .93 374
20	.87 652 .75 251	20	.91 573 .82 362	20	.94 689 .88 489	20	.97 056 .93 446
30	.87 724 .75 377	30	.91 631 .82 472	30	.94 734 .88 581	30	.97 089 .93 518
40	.87 796 .75 502	40	.91 689 .82 583	40	.94 779 .88 674	40	.97 123 .93 589
50	.87 868 .75 627	50	.91 747 .82 693	50	.94 824 .88 766	50	.97 156 .93 660
121	.87 939 .75 752	131	.91 805 .82 803	141	.94 869 .88 857	151	.97 188 .93 731
10	.88 011 .75 876	10	.91 862 .82 913	10	.94 914 .88 949	10	.97 221 .93 801
20	.88 082 .76 001	20	.91 919 .83 022	20	.94 958 .89 040	20	.97 253 .93 871
30	.88 153 .76 125	30	.91 976 .83 131	30	.95 003 .89 130	30	.97 285 .93 941
40	.88 223 .76 249	40	.92 033 .83 240	40	.95 047 .89 221	40	.97 317 .94 010
50	.88 294 .76 373	50	.92 090 .83 348	50	.95 090 .89 311	50	.97 349 .94 079
122	.88 364 .76 496	132	.92 146 .83 457	142	.95 134 .89 401	152	.97 381 .94 147
10	.88 434 .76 619	10	.92 202 .83 564	10	.95 177 .89 490	10	.97 412 .94 215
20	.88 503 .76 742	20	.92 258 .83 672	20	.95 221 .89 579	20	.97 443 .94 283
30	.88 573 .76 865	30	.92 314 .83 780	30	.95 264 .89 668	30	.97 474 .94 351
40	.88 642 .76 988	40	.92 369 .83 887	40	.95 306 .89 756	40	.97 505 .94 418
50	.88 711 .77 110	50	.92 425 .83 993	50	.95 349 .89 844	50	.97 536 .94 484
123	.88 780 .77 232	133	.92 480 .84 100	143	.95 391 .89 932	153	.97 566 .94 550
10	.88 848 .77 354	10	.92 534 .84 206	10	.95 433 .90 019	10	.97 597 .94 616
20	.88 916 .77 475	20	.92 589 .84 312	20	.95 475 .90 106	20	.97 627 .94 682
30	.88 984 .77 597	30	.92 643 .84 418	30	.95 517 .90 193	30	.97 656 .94 747
40	.89 052 .77 718	40	.92 698 .84 523	40	.95 559 .90 279	40	.97 686 .94 811
50	.89 120 .77 839	50	.92 751 .84 628	50	.95 600 .90 365	50	.97 716 .94 876
124	.89 187 .77 960	134	.92 805 .84 733	144	.95 641 .90 451	154	.97 745 .94 940
10	.89 254 .78 080	10	.92 859 .84 837	10	.95 682 .90 537	10	.97 774 .95 003
20	.89 321 .78 200	20	.92 912 .84 942	20	.95 723 .90 621	20	.97 803 .95 066
30	.89 387 .78 320	30	.92 965 .85 045	30	.95 763 .90 706	30	.97 831 .95 129
40	.89 454 .78 440	40	.93 018 .85 149	40	.95 804 .90 790	40	.97 860 .95 192
50	.89 520 .78 560	50	.93 071 .85 252	50	.95 844 .90 874	50	.97 888 .95 254
125	.89 586 .78 679	135	.93 123 .85 355	145	.95 884 .90 958	155	.97 916 .95 315
10	.89 651 .78 798	10	.93 175 .85 458	10	.95 924 .91 041	10	.97 944 .95 377
20	.89 717 .78 917	20	.93 227 .85 560	20	.95 963 .91 124	20	.97 972 .95 438
30	.89 782 .79 035	30	.93 279 .85 663	30	.96 002 .91 206	30	.97 999 .95 498
40	.89 847 .79 153	40	.93 331 .85 764	40	.96 042 .91 289	40	.98 027 .95 558
50	.89 912 .79 271	50	.93 382 .85 866	50	.96 081 .91 370	50	.98 054 .95 618
126	.89 976 .79 389	136	.93 433 .85 967	146	.96 119 .91 452	156	.98 081 .95 677
10	.90 040 .79 507	10	.93 484 .86 068	10	.96 158 .91 533	10	.98 108 .95 736
20	.90 104 .79 624	20	.93 535 .86 168	20	.96 196 .91 614	20	.98 134 .95 795
30	.90 168 .79 741	30	.93 585 .86 269	30	.96 234 .91 694	30	.98 161 .95 853
40	.90 232 .79 858	40	.93 636 .86 369	40	.96 272 .91 774	40	.98 187 .95 911
50	.90 295 .79 974	50	.93 686 .86 468	50	.96 310 .91 854	50	.98 213 .95 968
127	.90 358 .80 091	137	.93 736 .86 568	147	.96 347 .91 934	157	.98 239 .96 025
10	.90 421 .80 207	10	.93 785 .86 667	10	.96 385 .92 013	10	.98 264 .96 082
20	.90 484 .80 323	20	.93 835 .86 765	20	.96 422 .92 091	20	.98 290 .96 138
30	.90 546 .80 438	30	.93 884 .86 864	30	.96 459 .92 170	30	.98 315 .96 194
40	.90 608 .80 553	40	.93 933 .86 962	40	.96 495 .92 248	40	.98 340 .96 249
50	.90 670 .80 668	50	.93 982 .87 060	50	.96 532 .92 325	50	.98 365 .96 305
128	.90 732 .80 783	138	.94 030 .87 157	148	.96 568 .92 402	158	.98 389 .96 359
10	.90 794 .80 898	10	.94 079 .87 254	10	.96 604 .92 479	10	.98 414 .96 413
20	.90 855 .81 012	20	.94 127 .87 351	20	.96 640 .92 556	20	.98 438 .96 467
30	.90 916 .81 126	30	.94 175 .87 448	30	.96 676 .92 632	30	.98 462 .96 521
40	.90 977 .81 239	40	.94 223 .87 544	40	.96 712 .92 708	40	.98 486 .96 574
50	.91 037 .81 353	50	.94 270 .87 640	50	.96 747 .92 783	50	.98 510 .96 627
129	.91 098 .81 466	139	.94 318 .87 735	149	.96 782 .92 858	159	.98 533 .96 679
10	.91 158 .81 579	10	.94 365 .87 831	10	.96 817 .92 933	10	.98 557 .96 731
20	.91 218 .81 692	20	.94 412 .87 926	20	.96 852 .93 007	20	.98 580 .96 782
30	.91 277 .81 804	30	.94 458 .88 020	30	.96 886 .93 081	30	.98 603 .96 834
40	.91 337 .81 916	40	.94 505 .88 115	40	.96 921 .93 155	40	.98 625 .96 884
50	.91 396 .82 028	50	.94 551 .88 209	50	.96 955 .93 228	50	.98 648 .96 935
130	.91 455 .82 139	140	.94 597 .88 302	150	.96 989 .93 301	160	.98 670 .96 985

° /	θ L hav	Nath hav	° /	θ L hav	Nath hav
160	.98 670	.96 985	170	.99 669	.99 240
10	.98 692	.97 034	10	.99 680	.99 265
20	.98 714	.97 083	20	.99 691	.99 290
30	.98 736	.97 132	30	.99 701	.99 314
40	.98 758	.97 180	40	.99 712	.99 338
50	.98 779	.97 228	50	.99 722	.99 361
161	.98 801	.97 276	171	.99 732	.99 384
10	.98 822	.97 323	10	.99 742	.99 407
20	.98 842	.97 370	20	.99 751	.99 429
30	.98 863	.97 416	30	.99 761	.99 451
40	.98 884	.97 462	40	.99 770	.99 472
50	.98 904	.97 508	50	.99 779	.99 493
162	.98 924	.97 553	172	.99 788	.99 513
10	.98 944	.97 598	10	.99 797	.99 533
20	.98 964	.97 642	20	.99 805	.99 553
30	.98 983	.97 686	30	.99 814	.99 572
40	.99 003	.97 729	40	.99 822	.99 591
50	.99 022	.97 773	50	.99 830	.99 609
163	.99 041	.97 815	173	.99 838	.99 627
10	.99 059	.97 858	10	.99 845	.99 645
20	.99 078	.97 899	20	.99 853	.99 662
30	.99 096	.97 941	30	.99 860	.99 679
40	.99 115	.97 982	40	.99 867	.99 695
50	.99 133	.98 023	50	.99 874	.99 711
164	.99 151	.98 063	174	.99 881	.99 726
10	.99 168	.98 103	10	.99 887	.99 741
20	.99 186	.98 142	20	.99 894	.99 756
30	.99 203	.98 182	30	.99 900	.99 770
40	.99 220	.98 220	40	.99 906	.99 784
50	.99 237	.98 258	50	.99 912	.99 797
165	.99 254	.98 296	175	.99 917	.99 810
10	.99 270	.98 334	10	.99 923	.99 822
20	.99 287	.98 371	20	.99 928	.99 834
30	.99 303	.98 407	30	.99 933	.99 846
40	.99 319	.98 444	40	.99 938	.99 857
50	.99 335	.98 479	50	.99 943	.99 868
166	.99 350	.98 515	176	.99 947	.99 878
10	.99 366	.98 550	10	.99 951	.99 888
20	.99 381	.98 584	20	.99 955	.99 898
30	.99 396	.98 619	30	.99 959	.99 907
40	.99 411	.98 652	40	.99 963	.99 915
50	.99 425	.98 686	50	.99 967	.99 924
167	.99 440	.98 719	177	.99 970	.99 931
10	.99 454	.98 751	10	.99 973	.99 939
20	.99 468	.98 783	20	.99 976	.99 946
30	.99 482	.98 815	30	.99 979	.99 952
40	.99 496	.98 846	40	.99 982	.99 959
50	.99 510	.98 877	50	.99 984	.99 964
168	.99 523	.98 907	178	.99 987	.99 970
10	.99 536	.98 937	10	.99 989	.99 974
20	.99 549	.98 967	20	.99 991	.99 979
30	.99 562	.98 996	30	.99 993	.99 983
40	.99 575	.99 025	40	.99 994	.99 986
50	.99 587	.99 053	50	.99 996	.99 990
169	.99 599	.99 081	179	.99 997	.99 992
10	.99 611	.99 109	10	.99 998	.99 995
20	.99 623	.99 136	20	.99 999	.99 997
30	.99 635	.99 163	30	.99 999	.99 998
40	.99 646	.99 189	40	.00 000	.99 999
50	.99 658	.99 215	50	.00 000	1.00 000
170	.99 669	.99 240	180	.00 000	1.00 000

